EC 370
Intermediate Microeconomics II
Instructor: Sharif F. Khan

Department of Economics
Wilfrid Laurier University
Spring 2008

Suggested Solutions to Assignment 4 (OPTIONAL)

Total Marks: 40

Part B Problem Solving Questions [40 Marks]

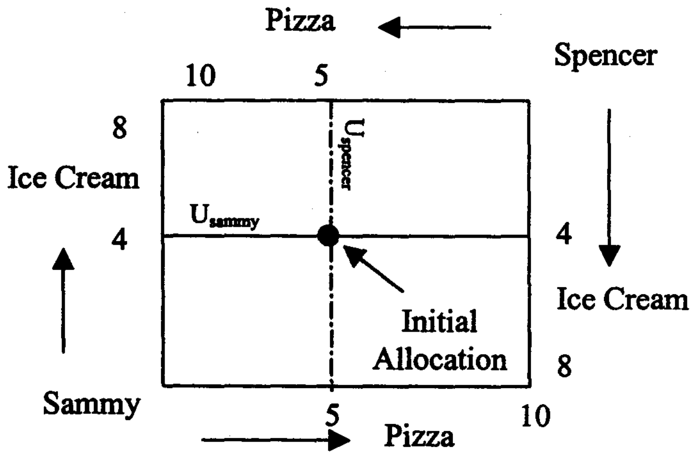
Read each part of the question very carefully. Show all the steps of your calculations to get full marks.

B1. a)

For an allocation to be efficient, it must be the case that we cannot reallocate the goods without making someone worse off. In this case, any reallocation that takes pizza away from Sammy and gives some to Spencer increases Spencer's utility without lowering Sammy's utility. Any reallocation that takes ice cream away from Spencer and gives some to Sammy increases Sammy's utility without lowering Spencer's utility. Since this type of reallocation is possible, the initial allocation is not efficient.

b) Draw the Edgeworth box, the initial allocation, and the indifference curves for Sammy and Spencer.

Answer



c) Identify the contract curve.

Answer

The contract curve shows all allocations of goods in the Edgeworth box that are economically efficient. Any allocation that gives any pizza to Sammy or that gives any ice cream to Spencer is not efficient, because giving pizza to Spencer raises Spencer's utility without lowering Sammy's utility, and giving ice cream to Sammy raises Sammy's utility without lowering Spencer's utility. Therefore, the contract curve consists of a single point, where Sammy has all 8 gallons of ice cream and 0 pizzas and Spencer has all 10 pizzas and 0 gallons of ice cream.

B2.

(a)

$$F = 6 L_F \quad \text{—— ①}$$

$$C = 5 (L_C)^{\frac{1}{2}} \quad \text{—— ②}$$

It is given that Robinson decides to work 6 hours per day. So,

$$L_C + L_F = 6 \quad \text{—— ③} \quad \text{must hold.}$$

Using equation ①,

$$L_F = \frac{F}{6} \quad \text{—— ①'}$$

Using equation ②,

$$C = 5 (L_C)^{\frac{1}{2}}$$

$$\Rightarrow \frac{C}{5} = (L_C)^{\frac{1}{2}}$$

$$\Rightarrow L_C = \frac{C^2}{25} \quad \text{—— ②'}$$

Substituting ①' & ②' into ③:

$$\frac{C^2}{25} + \frac{F}{6} = 6 \quad \text{—— ④}$$

④ is the equation of the PPF.

We can rewrite ④ as,

$$T(F, C) \equiv \frac{F}{6} + \frac{C^2}{25} - 6 = 0$$

By total differentiating both sides of the above equation:

$$\frac{\partial T(F,C)}{\partial F} . dF + \frac{\partial T(F,C)}{\partial C} . dC = 0$$

$$\Rightarrow \frac{1}{6} . dF + \frac{2C}{25} . dC = 0$$

$$\cancel{\Rightarrow \frac{2C}{25} \frac{dC}{dC} = - \frac{1}{6} dF}$$

$$\Rightarrow \frac{2C}{25} dC = -\frac{1}{6} dF$$

$$\Rightarrow \frac{dC}{dF} = -\frac{\frac{1}{6}}{\frac{2C}{25}} = -\frac{25}{12C}$$

$$\text{So, } MRT_{F,C} \equiv \frac{dC}{dF} = -\frac{25}{12C} < 0 \text{ ,}$$

$$\frac{\partial MRT}{\partial C} = -(-1)\frac{25C^{-2}}{12} = \frac{25}{12C^2} > 0$$

THIS MEANS THAT THE ALGEBRAIC VALUE OF THE SLOPE OF THE PPF INCREASES AS THE ECONOMY PRODUCES MORE COCONUTS. THUS, MRT shows increasing opportunity cost to specialization. That means the PPF will be downward ~~Slope~~ sloping and concave to the origin.

To find the horizontal intercept, set $C=0$ into (4):

$$\frac{F}{6} = 6$$

$$\therefore F = 36$$

To find the vertical intercept, set $F=0$ into (4):

$$\frac{C^2}{25} = 6$$

$$\Rightarrow C^2 = 150$$

$$\therefore C = \sqrt{150} = 12.25$$

Figure B2 illustrates the PPF of this economy.

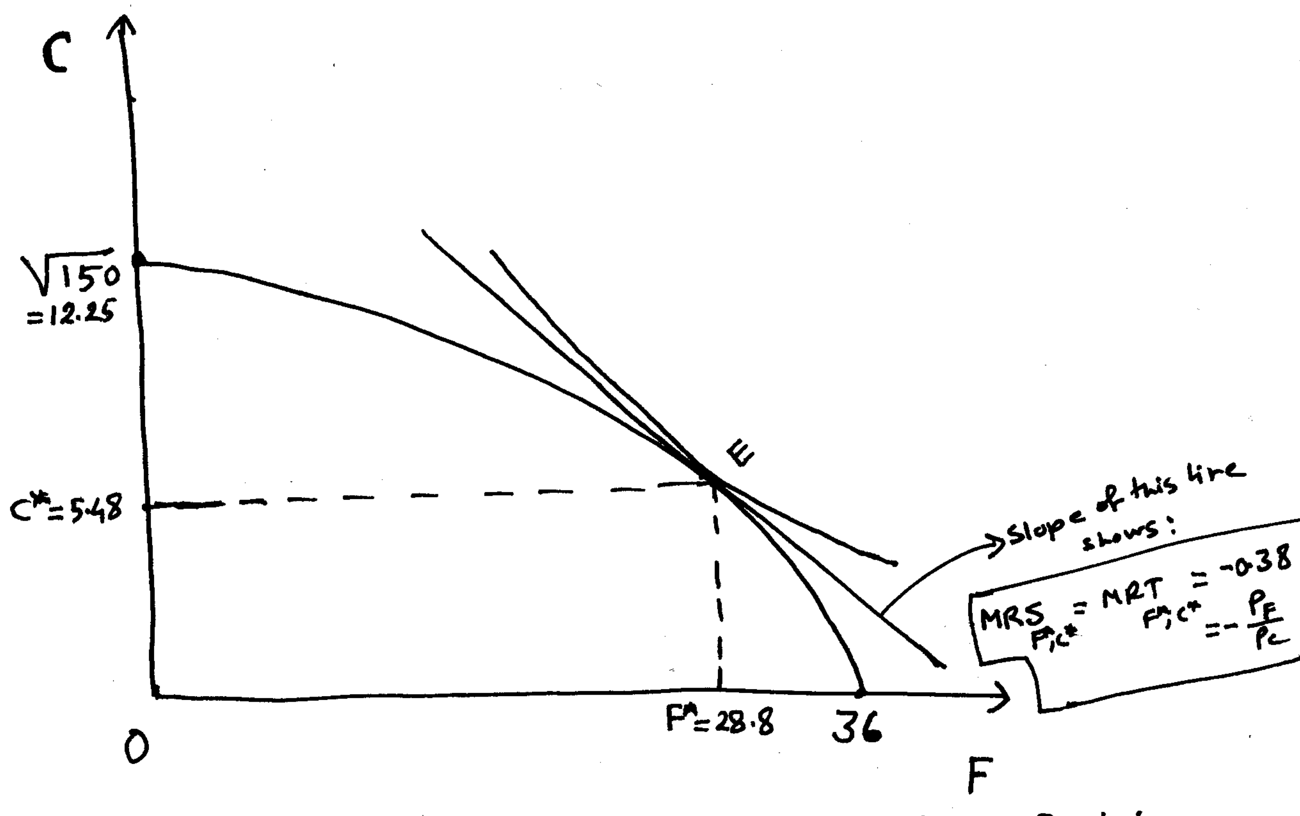


FIGURE B2: THE UTILITY MAXIMIZING CHOICE IN Robinson Crusoe's economy.

(b)

Robinson Crusoe will maximize his utility subject to the resource constraint of his economy which is given by the PPF equation.

$$\max_{\{F, C\}} \quad U = F^2 C$$

$$\text{s.t.} \quad \frac{F}{6} + \frac{C^2}{25} = 6$$

$$\mathcal{L} = F^2 C + \lambda \left[\frac{F}{6} + \frac{C^2}{25} - 6 \right]$$

F.O.N.Cs :

$$\frac{\partial \mathcal{L}}{\partial F} : \quad 2FC + \frac{\lambda}{6} = 0 \Rightarrow \lambda = -12FC \quad \text{——} (5)$$

$$\frac{\partial \mathcal{L}}{\partial C} : \quad F^2 + \lambda \frac{2C}{25} = 0 \Rightarrow F^2 = -\frac{\lambda 2C}{25} \quad \text{——} (6)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} : \quad \frac{F}{6} + \frac{C^2}{25} - 6 = 0 \Rightarrow \frac{F}{6} + \frac{C^2}{25} = 6 \quad \text{——} (7)$$

Substituting (5) into (6):

$$F^2 = \frac{24FC^2}{25}$$

$$\Rightarrow F = \frac{24C^2}{25} \quad \text{——} (8)$$

Substituting (8) into (7):

$$\left(\frac{24C^2}{25}\right)\frac{1}{6} + \frac{C^2}{25} = 6$$

$$\Rightarrow \frac{4C^2}{25} + \frac{C^2}{25} = 6$$

$$\Rightarrow \frac{5C^2}{25} = 6$$

$$\Rightarrow \frac{C^2}{5} = 6$$

$$\Rightarrow C^2 = 30$$

$$\therefore C^* = \pm\sqrt{30}$$

We will only consider the positive value of C:

$$C^* = +\sqrt{30} = 5.48$$

Substituting $C^* = +\sqrt{30}$ into (8):

$$F^* = \frac{24(C^*)^2}{25} = \frac{24(\sqrt{30})^2}{25} = \frac{24(30)}{25}$$

$$= 28.8$$

The point E on Figure B2 illustrates this utility-maximizing combination of F and C : $(F^*, C^*) = (28.8, 5.48)$.

(c)

$$MRS_{F,C} \equiv - \frac{\frac{\partial U}{\partial F}}{\frac{\partial U}{\partial C}} = - \frac{2FC}{F^2} = - \frac{2C}{F}$$

At the ~~equili~~ utility maximizing choice,

$$MRS_{F^*,C^*} = - \frac{2C^*}{F^*} = - \frac{2\sqrt{30}}{\frac{24(30)}{25}} = - \frac{50}{24\sqrt{30}}$$

$$= - \frac{25}{12\sqrt{30}}.$$

So, $MRS_{F^*,C^*} = - \frac{25}{12\sqrt{30}}$

At the utility maximizing choice,

$$MRT_{F^*,C^*} = - \frac{25}{12C^*} = - \frac{25}{12\sqrt{30}}$$

So, at the utility maximizing choice,

$$MRS_{F^*,C^*} = - \frac{25}{12\sqrt{30}} = MRT_{F^*,C^*}$$ [Proved].

(d) The implicit price ratio between fish and cocunuts $\left(\frac{P_F}{P_C}\right)$ at the utility maximizing choice:

$$\frac{P_F}{P_C} = |MRS_{F^*,C^*}| = |MRT_{F^*,C^*}| = \left| - \frac{25}{12\sqrt{30}} \right| = \frac{25}{12\sqrt{30}}$$

$$= 0.38$$

So, at the utility maximizing choice the price of 1 unit of fish is 0.38 units of ~~the~~ cocanuts.