Suggested Solutions to Assignment 3 (Optional)

Total Marks: 90

Problem Solving Questions

Read each part of the questions very carefully. Show all the steps of your calculations to get full marks.

B1. [15 Marks]

Suppose market demand is $P = 130 - Q$.

(a) If two firms compete in this market with constant marginal and average costs, $c = 10$, find the Cournot equilibrium output and profit per firm.

Suppose firm 1 takes firm 2's output choice q_2 as given. Then firm 1's problem is to maximize its profit by choosing its output level q_1. If firm 1 produces q_1 units and firm 2 produces q_2 units then total quantity supplied is $q_1 + q_2$. Define $Q \equiv q_1 + q_2$. The market price will be $P = 130 - q_1 - q_2$.

Firm 1's profit maximization problem:

$$\max_{q_1} \ \pi_1(q_1, q_2) = [130 - (q_1 + q_2)]q_1 - 10q_1$$

First order conditions:

$$\frac{\partial \pi_1}{\partial q_1} = 130 - (q_1 + q_2) + q_1(-1) - 10 = 0$$

$$\Rightarrow 120 - 2q_1 - q_2 = 0$$
$$\Rightarrow 2q_1 = 120 - q_2$$
$$\Rightarrow q_1 = \frac{120 - q_2}{2}$$

So, Firm 1's best response to q_2 or Firm 1's reaction function is:

$$q_1 = R(q_2) = \frac{120 - q_2}{2} \qquad (1)$$

Since the profit- maximization problem faced by the two firms are symmetric in this case, Firm 2's best response to q_1 or Firm 2's reaction function will have the same functional form as (1):

$$q_2 = R(q_1) = \frac{120 - q_1}{2} \qquad (2)$$

The symmetry of the problem also implies that at the Cournot-Nash equilibrium both firms will produce the same level of output:

$$q_1^* = q_2^* = q^* \qquad (3)$$

Substituting $q_1 = q_2 = q^*$ into either (1) or (2) and solving for q^* we get:

$$\begin{aligned} q^* &= \frac{120 - q^*}{2} \\ \Rightarrow 2q^* &= 120 - q^* \\ \Rightarrow 3q^* &= 120 \\ \Rightarrow q^* &= 40 \end{aligned}$$

So, the Cournot-Nash equilibrium output is:

$$(q_1^*, q_2^*) = (40,40)$$

Firm 1's profit at the equilibrium: $\pi_1(q_1^*, q_2^*) = [130 - (q_1^* + q_2^*)]q_1^* - 10q_1^*$

$$\begin{aligned} &= [130 - (40 + 40)]40 - 10(40) \\ &= 2000 - 400 \\ &= 1600 \end{aligned}$$

Because of the symmetry of the problem Firm 2's profit is same as Firm 1's profit at the equilibrium:

$$\pi_2(q_1^*, q_2^*) = \pi_1(q_1^*, q_2^*) = 1600.$$

(b) Find the monopoly output and profit if there is only one firm with marginal cost $c = 10$.

The monopolist's problem is to maximize profit by choosing its output level, *Q*. The profit-maximization problem of the monopolist:

$$\max_{Q} \ \pi_m(Q) = (130 - Q)Q - 10Q$$

First order conditions:

$$\frac{\partial \pi_m}{\partial Q} = 130 - Q + Q(-1) - 10 = 0$$
$$\Rightarrow 120 - 2Q = 0$$
$$\Rightarrow 2Q = 120$$
$$\Rightarrow Q = 60$$

So, the profit-maximizing equilibrium output of the monopolist is: $Q_m^* = 60$.

The profit-maximizing equilibrium price of the monopolist is:
$P_m^* = 130 - Q_m^* = 130 - 60 = 70$.

Equilibrium profit of the monopolist is: $\pi_m^* = (P_m^* - c)Q_m^* = (70 - 10)60 = 3600$.

(c) Using the information from parts (a) and (b), construct a 2×2 payoff matrix where the strategies available to each of two players are to produce the Cournot equilibrium quantity or half the monopoly quantity.

When both firms choose the Cournot equilibrium quantity, each earns the Cournot equilibrium profit which is calculated in part (a). Similarly, producing half the monopoly output garners each firm half the monopoly profit: $\pi_1 = \pi_2 = \frac{1}{2}\pi_m^* = \frac{1}{2}(3600) = 1800$.

When, for instance, firm 1 produces the Cournot output, $q_1 = 40$, while firm 2 produces half the monopoly output, $q_2 = \frac{1}{2}Q_m^* = \frac{1}{2}(60) = 30$, the market price would be $P = 130 - (40 + 30) = 60$. Firm 1 earns $\pi_1 = (P - c)q_1 = (60 - 10)40 = 2000$ and firm 2 earns $\pi_2 = (P - c)q_2 = (60 - 10)30 = 1500$. When the roles are reversed, so are profits.

The payoff of matrix of a game where each of the two firms can choose between the half the monopoly quantity (3) or the Cournot equilibrium quantity (40) is as follows:

		Firm2	
		$q_2 = 30$	$q_2 = 40$
Firm 1	$q_1 = 30$	1800, 1800	1500, 2000
	$q_1 = 40$	2000, 1500	1600, 1600

(d) What is the Nash equilibrium (or equilibria) of the game you constructed in part (c)? Is there any mixed strategy Nash equilibrium in this game? If yes, what is the mixed strategy Nash equilibrium (or equilibria)?

The game in part (c) is a typical Prisoner's Dilemma game, where producing the Cournot equilibrium output, i.e., choosing $q = 40$, is the dominant strategy for each firm. If Firm 1 chooses $q_1 = 30$, the best response for Firm 2 is to choose $q_2 = 40$ because $2000 > 1800$. If Firm 1 chooses $q_2 = 40$, the best response for Firm 2 is to choose $q_2 = 40$ because $1600 > 1500$. Thus, $q_2 = 40$ is the dominant strategy for Firm 2. Since the game is symmetric, we can argue that $q_1 = 40$ is also the dominant strategy for Firm 2. This game has a unique Nash equilibrium where each firm plays its dominant strategy. That is, $(q_1 = 40, q_2 = 40)$ is the unique Nash equilibrium of this game. Note that this is a pure strategy Nash equilibrium.

In this game there is no mixed strategy Nash equilibrium because each firm has a dominant strategy which is to produce the Cournot equilibrium output. Given that firm 1 is choosing $q_1 = 40$ with a 100 percent probability, the best response of firm 2 is to choose $q_2 = 40$ with a 100 percent probability rather than to choose a randomized strategy over $q_2 = 30$ and $q_2 = 40$, vice versa.

B2. [15 Marks]

Consider a Rubenstein bargaining game between two players, Alan and David. They have $5 to divide between them. They agree to spend at most four days negotiating over the division. The first day, Alan will make an offer, David either accepts or comes back with a counteroffer the next day, and on the fourth day David gets to make one final offer. If they cannot reach an agreement in four days, both players get zero.

We assume Alan and David differ in their degree of impatience: Alan's discount factor is α per day and David's discount factor is β per day. We also assume that if a player is indifferent between two offers, he will accept the one that is most preferred by his opponent.

(a) Draw the extensive form of this bargaining game.

See Figure B2(a).

(b) Find the subgame perfect equilibrium of this bargaining game.

See Figure B2(b).

The subgame perfect equilibrium of this bargaining game is:

In period 1, Alan offers $X_1 = 5 - \beta[5 - \alpha(5 - 5\beta)]$ to himself and $5 - X_1 = \beta[5 - \alpha(5 - 5\beta)]$ to David. David accepts the offer.

In period 2, David offers $X_2 = \alpha(5 - 5\beta)$ to Alan and $5 - X_2 = 5 - \alpha(5 - 5\beta)$ to himself. Alan accepts the offer.

In period 3, Alan offers $X_3 = 5 - 5\beta$ to himself and $5 - X_3 = 5\beta$ to David. David accepts the offer.

In period 4, David offers $X_4 = 0$ to Alan and $5 - X_4 = 5$ to himself. Alan accepts the offer.

[Note: See Chapter 29 Lecture Slides on strategic bargaining (slides # 181 to 199) to understand how to derive the subgame perfect equilibrium offers by backward induction]

(c) What is the final outcome of this game?

Alan and David reach an agreement at the end of the first day. In period 1, Alan offers $X_1 = 5 - \beta[5 - \alpha(5 - 5\beta)]$ to himself and $5 - X_1 = \beta[5 - \alpha(5 - 5\beta)]$ to David. David accepts the offer. So, the game ends after the first day of bargaining.

(d) If David becomes more impatient what will happen to the equilibrium payoff to Alan?

If David becomes more impatient, he will discount his future payoff at a higher rate. In other words, the value of David's discount factor, β, decreases as he becomes more impatient.

Alan gets $5-\beta[5-\alpha(5-5\beta)]$ as a final outcome of this game at the end of first day. The value of his payoff increases with a decrease in β, all other things remaining constant.

B3. [15 Marks]

Two firms are competing in an oligopolistic industry. Firm 1, the larger of the two firms, is contemplating its capacity strategy, which could be either "aggressive" or "passive". The aggressive strategy involves a large increase in capacity aimed at increasing the firm's market share, while the passive strategy involves no change in the firm's capacity. Firm 2, the smaller competitor, is also pondering its capacity expansion strategy; it will also choose between an aggressive strategy and a passive strategy. The table below shows the profits associated with each pair of choices.

		Firm2	
		Aggressive	**Passive**
Firm 1	**Aggressive**	**25, 9**	**33, 10**
	Passive	**30, 13**	**36, 12**

(a) If both firms decide their strategies simultaneously, what is the Nash equilibrium (or equilibria)? Is there any mixed strategy Nash equilibrium in this game? If yes, what is the mixed strategy Nash equilibrium (or equilibria)?

If Firm 2 chooses "aggressive", the best response for Firm 1 is to choose "passive". Because $30 > 25$. If Firm 2 chooses "passive", the best response for Firm 1 is to choose "passive". Because $36 > 33$. This implies that "passive" is a dominant strategy for Firm 1. However, there is no dominant strategy for Firm 2 in this game.

Firm 1 will choose its dominant strategy "passive". Firm 2, knowing 1 firm 1 has a dominant strategy, will play its best response, "aggressive". This is the only Nash equilibrium in the simultaneous-move game.

There is no mixed strategy Nash equilibrium because one of the players, Firm 1, has a dominant strategy in this game. Given that firm 1 is choosing "passive" with a 100 percent probability, the best response of firm 2 is to choose "aggressive" with a 100 percent probability rather than to choose a randomized strategy over "passive" and "aggressive", vice versa.

Now assume that Firm 1 can decide first and can credibly commit to its capacity expansion strategy.

(b) Draw the extensive form of this sequential game. What is the subgame perfect equilibrium? What is the final outcome of this game?

See Figure B3(b) for the extensive form representation of this game.

As shown in Figure B3(b), if Firm 1 can choose first, then if it chooses "aggressive" Firm 2 will choose "passive" and Firm 1 will receive 33. If Firm 1 instead chooses "passive", then Firm 2 will select "aggressive" and Firm 1 will receive a payoff of 30. Therefore, if Firm 1 can move first, it does best to select "aggressive" in which case Firm 2 will select its best response "passive" earning Firm 1 a payoff of 33 and Firm 2 a payoff of 10.

The Subgame perfect equilibrium:

In period 2 Firm 2 will choose "passive" if Firm 1 chooses "aggressive" in period 1.

In period 2 Firm 2 will choose "aggressive" if Firm 1 chooses "passive" in period 1.

In period 1 Firm 1 will choose "aggressive".

The final outcome of this game:

In period 1 Firm 1 will choose "aggressive" and in period 2 Firm 1 will choose "passive" earning Firm 1 a payoff of 33 and Firm 2 a payoff of 10.

(c) If Firm 2 threatens to play "aggressive" if Firm 1 plays "aggressive", will it be credible to Firm 1? If this threat is not credible to Firm 1, what could Firm 2 do to make its threat credible?

If firm 2 threatens to play "aggressive" if Firm 1 plays "aggressive", it will NOT be credible to Firm 1. Because Firm 1 knows that if it plays "aggressive" in period 1, Firm 2's best response in period 2 will be "passive". Because $10 > 9$.

Firm 2's problem is that once Firm 1 has made its choice, Firm 1 expects Firm 2 to do the rational thing. Firm 2 can make its threat credible if it could commit itself to play would be better off if it could ***commit*** itself to play "aggressive" if Firm 1 plays "aggressive".

One way for Firm 2 to make such a commitment is to allow someone else to make its choices. For example, Firm 2 might hire a lawyer and instruct him to play "aggressive" if Firm 1 plays "aggressive". If Firm 1 is aware of these instructions, the situation is radically different from its point of view. If Firm 1 knows about Firm 2's instructions to its lawyer, then it knows that if it plays "aggressive" it will end up with a payoff of 25.

So, the sensible thing for Firm 1 to do is to play "passive" and get a relatively higher payoff of 30. See Figure B3(c) for an extensive form representation of this case.

B5. [15 Marks]

Suppose demand for a commodity is given by $y = 100 - p$. There are only two possible factories that can produce this commodity, each with cost function: $c_j = 50 + y_j^2$, where $j = 1, 2$ denotes the factory. The total market output is the sum of the outputs from these two plants.

(a) **Find the efficient level of output and price for this market. Also, find the total profits of the two firms in this situation**.

The total market output is the sum of the outputs from these two plants. That is,
$y = y_1 + y_2$

Market Demand: $p = 100 - (y_1 + y_2)$
Factory 1's cost function: $c_1 = 50 + y_1^2$
Factory 1's marginal cost function: $MC_1 = \frac{\partial c_1}{\partial y_1} = 2y_1$
Factory 2's cost function: $c_2 = 50 + y_2^2$
Factory 2's marginal cost function: $MC_2 = \frac{\partial c_2}{\partial y_2} = 2y_2$

At the efficient level of output of the following condition has to be satisfied:
$P = MC_1 = MC_2$

This means that the following two equations have to be satisfied:

$100 - y_1 - y_2 = 2y_1$ (1)

and

$100 - y_1 - y_2 = 2y_2$ (2)

Since both factories have the same cost function, at the efficient level of production both must produce exactly the same level of output:

$y_1^e = y_2^e$

Setting $y_1 = y_2$ into equation (1) we get:

$100 - y_2 - y_2 = 2y_2$
$\Rightarrow 4y_2 = 100$
$\Rightarrow y_2^e = 25$

So, $y_1^e = 25$

The efficient level of total output: $y^e = y_1^e + y_2^e = 25 + 25 = 50$

Factory 1's profit: $\pi_1^e = p^e y_1^e - 50 - \left(y_1^e\right)^2 = 50(25) - 50 - (25)^2 = 575$
Factory 2's profit: $\pi_1^e = \pi_2^e = 575$

Total profit of the two factories: $\pi^e = \pi_1^e + \pi_2^e = 575 + 575 = 1150$

(b) Suppose the two firms form a cartel. Compute the profit maximizing total output, pice, profits, and deadweight loss of the cartel in this situation.

The cartel's problem is to maximize the joint profits by choosing y_1 and y_2.

$$\max_{\{y_1,\, y_2\}} \quad \pi^m = (100 - y_1 - y_2)(y_1 + y_2) - 50 - y_1^2 - 50 - y_2^2$$

First order conditions:

$$\frac{\partial \pi^m}{\partial y_1} = 100 - 2y_1 - y_2 - y_2 - 2y_1 = 0$$

$$\Rightarrow 4y_1 = 100 - 2y_2 \qquad (3)$$

$$\frac{\partial \pi^m}{\partial y_2} = 100 - y_1 - 2y_2 - y_1 - 2y_2 = 0$$

$$\Rightarrow 4y_2 = 100 - 2y_1 \qquad (4)$$

Since both factories have the same cost function, at the joint-profit maximizing equilibrium they will produce the same amount of output.

$$y_1^m = y_2^m$$

Setting $y_1 = y_2$ into equation (3) we get:

$$4y_1 = 100 - 2y_1$$
$$\Rightarrow 6y_1 = 100$$
$$\Rightarrow y_1^m = \frac{100}{6}$$

So, $y_2^m = y_1^m = \frac{100}{6}$.

The joint-profit maximizing total output of the cartel,

$$y_2^m = y_1^m + y_2^m = \frac{100}{6} + \frac{100}{6} = \frac{100}{3} = 33.33$$

The joint-profit maximizing total output of the cartel,

$$p^m = 100 - (y_1^m + y_2^m) = 100 - (33.33) = 66.67$$

The maximized profit of the cartel:

$$\pi^m = p^m\left(y^m\right) - 50 - \left(y_1^m\right)^2 - 50 - \left(y_2^m\right)^2 = 66.67(33.33) - 100 - 2\left(\frac{100}{6}\right)^2 = 1566.67$$

Deadweight loss of each factory under the cartel is the area between the downward sloping market demand curve and individual factory's upward sloping marginal cost curve over the quantities between the joint-profit maximizing output and the efficient level of output of each factory (try to visualize this area by drawing a diagram).

Deadweight loss of the cartel

= Deadweight loss of factory 1 + Deadweight loss of factory 2

$$= \frac{1}{2}\left(66.67 - \frac{100}{3}\right)\left(25 - \frac{100}{6}\right) + \frac{1}{2}\left(66.67 - \frac{100}{3}\right)\left(25 - \frac{100}{6}\right)$$

= (33.34)(8.33)

=277.7

(c) Instead of a cartel, suppose the two plants are owned by Cournot duopolists. Find the Cournot-Nash equilibrium output by each firm, the price, and the total profit. Also compute the deadweight loss associated with the Cournot duopoly.

You can solve the problem following the method used in solving question B1 (a). You can find the deadweight using the technique used in B5 (b). You can your answers with the following results:

The Cournot-Nash equilibrium output:

$$\left(y_1^c, y_2^c\right) = (20{,}20)$$

The market price at the Cournot-Nash equilibrium: $p^c = 60$

The total profit of the firms at the Cournot-Nash equilibrium: $\pi^c = 1500$

The total deadweight loss associated with the Cournot duopoly: $DWL^c = 100$

(d) Instead of the Cournot assumption, suppose that firm 1 sets its output before firm 2 does. Firm 2 does observes the output choice of firm 1 before it makes its own output choice. Find the Stackelberg equilibrium output produced by each firm, the price, and total profit. Also compute the deadweight loss in this situation.

In this Stackelberg game, Firm 1 is the quantity leader and Firm 2 is the quantity follower. We can solve this sequential game by backward induction. That means we have to first solve the follower's problem and then solve the leader's problem.

Firm 2's (the follower's) problem in period 2 is to maximize its profit by choosing y_2 for a given level of y_1 chosen by Firm 1 (the leader) in the first period.

Firm 2's profit maximization problem:

$$\max_{y_2} \ \pi_2^s(y_1, y_2) = [100 - (y_1 + y_2)]y_2 - 50 - (y_2)^2$$

First order conditions:

$$\frac{\partial \pi_2}{\partial y_2} = 100 - (y_1 + y_2) + y_2(-1) - 2y_2 = 0$$
$$\Rightarrow 4y_2 = 100 - y_1$$
$$\Rightarrow y_2 = \frac{100 - y_1}{4}$$

So, Firm 2's best response to y_1 or Firm 2's best response or reaction function is

$$y_2 = R(y_1) = \frac{100 - y_1}{4} \qquad (5)$$

Firm 1's (the leader's) problem in period 1 is to maximize its profit by choosing y_1 and considering the fact the best response function of Firm 2 in period 2 is given by equation

Firm 1's profit maximization problem:

$$\max\ \pi_1^s(y_1, y_2) = [100 - (y_1 + y_2)]y_1 - 50 - (y_1)^2$$
$$= [100 - (y_1 + R(y_1)]y_1 - 50 - (y_1)^2$$
$$= \left[100 - \left(y_1 + \frac{100 - y_1}{4}\right)\right]y_1 - 50 - (y_1)^2$$
$$= \left[100 - \left(\frac{100 + 3y_1}{4}\right)\right]y_1 - 50 - (y_1)^2$$
$$= \left[\frac{300 - 3y_1}{4}\right]y_1 - 50 - (y_1)^2$$

First order conditions:

$$\frac{\partial \pi_1^S}{\partial y_1} = \frac{300}{4} - \frac{6}{4}y_1 - 2y_1 = 0$$
$$\Rightarrow \frac{300 - 6y_1 - 8y_1}{4} = 0$$
$$\Rightarrow 300 - 6y_1 - 8y_1 = 0$$
$$\Rightarrow 14y_1 = 300$$
$$\Rightarrow y_1^s = 21.43$$

Substituting $y_1 = y_1^s = 21.43$ into Firm 2's best response or reaction function we get:

$$y_2^s = \frac{100 - y_1^s}{4} = \frac{100 - 21.43}{4} = 19.64 \qquad (5)$$

Therefore, the Stackelberg equilibrium quantities produced by each firm are:
$y_1^s = 21.43$ and $y_2^s = 19.64$.

The market price at the Stackelberg equilibrium:
$p^s = 100 - (y_1^s + y_2^s) = 100 - (21.43 + 19.64) = 58.93$

Firm 2's profit at the Stackelberg equilibrium:

$$\pi_2^s(y_1, y_2) = p^s y_2^s - 50 - (y_2^s)^2 = 58.93(19.64)\text{-}50\text{-}(19.64)^2 = 721.66$$

Firm 1's profit at the Stackelberg equilibrium:

$$\pi_1^s(y_1, y_2) = p^s y_1^s - 50 - (y_1^s)^2 = 58.93(21.43)\text{-}50\text{-}(21.43)^2 = 753.64$$

Total profits of the firms at the Stackelberg equilibrium:

$$\pi^s(y_1, y_2) = \pi_1^s(y_1, y_2) + \pi_2^s(y_1, y_2) = 753.64 + 721.66 = 1475.3$$

Deadweight loss of each factory in the Stackelberg model is the area between the downward sloping market demand curve and individual factory's upward sloping marginal cost curve over the quantities between the Stackelberg equilibrium output and the efficient level of output of each factory (try to visualize this area by drawing a diagram).

Total deadweight loss in the Stackelberg model

= Deadweight loss of factory 1 + Deadweight loss of factory 2

$= \frac{1}{2}(58.93 - 42.86)(25 - 21.43) + \frac{1}{2}(58.93 - 39.28)(25 - 19.64)$

= 28.68+52.66

=81.34

(e) Compare the results you found in (a), (b), (c), and (d).

Try to compare the results on your own.

B6. [15 Marks]

Two firms, Firm 1 and Firm 2, are competing in an oligopolistic industry. They produce an identical product. But Firm 1 does it at a lower cost than Firm 2. Firm 1 has a constant marginal cost of $15 and firm 2 has a constant marginal cost of $30. The market demand for the commodity is $p = 120 - y$, where y is aggregate output.

(a) Suppose that firms choose quantities. Find both best-response functions. Remember, marginal costs are different, so the best response functions will not be symmetric. Find the Cournot-Nash equilibrium quantities. Illustrate your results in a diagram.

Suppose firm 1 takes firm 2's output choice y_2 as given. Then firm 1's problem is to maximize its profit by choosing its output level y_1. If firm 1 produces y_1 units and firm 2 produces y_2 units then total quantity supplied is $y_1 + y_2$. Define $y \equiv y_1 + y_2$. The market price will be $P = 120 - y_1 - y_2$.

Firm 1 has a constant marginal and average cost of $15: $c_1 = 15$.

Firm 2 has a constant marginal and average cost of $30: $c_2 = 30$.

Firm 1's profit maximization problem:

$$\max_{y_1} \ \pi_1(y_1, y_2) = [120 - (y_1 + y_2)]y_1 - 15y_1$$

First order conditions:

$$\frac{\partial \pi_1}{\partial y_1} = 120 - (y_1 + y_2) + y_1(-1) - 15 = 0$$

$$\Rightarrow 105 - 2y_1 - y_2 = 0$$
$$\Rightarrow 2y_1 = 105 - y_2$$
$$\Rightarrow y_1 = \frac{105 - y_2}{2}$$

So, Firm 1's best response to y_2 or Firm 1's best response or reaction function is

:

$$y_1 = R(y_2) = \frac{105 - y_2}{2} \qquad (1)$$

Since the profit- maximization problem faced by the two firms are NOT symmetric in this case, we have to explicitly solve Firm 2's problem to find its best response function or reaction function.

Firm 2's profit maximization problem:

$$\max_{y_2} \; \pi_2(y_1, y_2) = [120 - (y_1 + y_2)]y_2 - 30y_2$$

First order conditions:

$$\frac{\partial \pi_2}{\partial y_2} = 120 - (y_1 + y_2) + y_2(-1) - 30 = 0$$

$$\Rightarrow 90 - y_1 - 2y_2 = 0$$
$$\Rightarrow 2y_2 = 90 - y_1$$
$$\Rightarrow y_2 = \frac{90 - y_1}{2}$$

So, Firm 2's best response to y_1 or Firm 2's best response or reaction function is

: $$y_2 = R(y_1) = \frac{90 - y_1}{2} \qquad (2)$$

To find the Cournot-Nash equilibrium quantities, we have to solve equation (1) and (2) simultaneously for y_1 and y_2.

Substituting (1) into (2),

$$y_2 = \frac{90}{2} - \frac{1}{2}\left(\frac{105 - y_2}{2}\right)$$
$$\Rightarrow y_2 = \frac{90}{2} - \frac{105}{4} + \frac{y_2}{4}$$
$$\Rightarrow y_2 - \frac{y_2}{4} = \frac{90}{2} - \frac{105}{4}$$
$$\Rightarrow \frac{3y_2}{4} = \frac{75}{4}$$
$$\Rightarrow 3y_2 = 75$$
$$\Rightarrow y_2^* = 25$$

Substituting $y_2 = y_2^* = 25$ into (1),

$$y_1^* = \frac{105 - y_2^*}{2} = \frac{105 - 25}{2} = 40\,.$$

So, the Cournot-Nash equilibrium quantities are:

$$\left(y_1^*, y_2^*\right) = (40{,}25)$$

The market price at the Cournot-Nash equilibrium is:

$$P^* = 120 - y_1^* - y_2^* = 120 - 40 - 25 = 55$$

(b) Suppose that the firms choose prices instead of quantities and that prices must be announced in dollars and cents. (That is, $15.71, and $39.00 are permissible prices, but $45.975 is not.) What are the Bertrand equilibrium prices? How much does each firm earn in the Bertrand equilibrium?

Because of the price competition between the two firms, the high-cost firm, Firm 2, will not be able to set a price higher than its marginal cost ($30). If Firm 2 sets a price higher than $30, the low-cost firm will undercut that price and capture the entire market. Thus, price competition will force the high-cost firm to set its price equal to 30, $P_2 = 30$.
The low-cost firm, Firm 1, will undercut this price by setting its price slightly less than 30 and capture the entire market. It will set its price equal to 29.99, $P_1 = 29.99$.

So, the Bertrand equilibrium prices are: $P_1 = 29.99$ and $P_2 = 30$.

At the equilibrium Firm 1 makes all the sales. Total units of output sold by Firm1:
$y_1^* = 120 - P_1^* = 120 - 29.99 = 90.01$

Firm 1's profits: $\pi_1^* = (P_1^* - c_1)\, y_1^* = (29.99 - 15)90.01 = 1349.25$

Firm 2's profits: $\pi_2^* = 0$