Assignment 3 (Optional)

Total Marks: 90

Problem Solving Questions

Read each part of the questions very carefully. Show all the steps of your calculations to get full marks.

B1. [15 Marks]

Suppose market demand is $P = 130 - Q$.

(a) If two firms compete in this market with constant marginal and average costs, $c = 10,$ find the Cournot equilibrium output and profit per firm.

(b) Find the monopoly output and profit if there is only one firm with marginal cost $c = 10$.

(c) Using the information from parts (a) and (b), construct a 2×2 payoff matrix where the strategies available to each of two players are to produce the Cournot equilibrium quantity or half the monopoly quantity.

(d) What is the Nash equilibrium (or equilibria) of the game you constructed in part **(c)**? Is there any mixed strategy Nash equilibrium in this game? If yes, what is the mixed strategy Nash equilibrium (or equilibria)?

B2. [15 Marks]

Consider a Rubenstein bargaining game between two players, Alan and David. They have $5 to divide between them. They agree to spend at most four days negotiating over the division. The first day, Alan will make an offer, David either accepts or comes back with a counteroffer the next day, and on the fourth day David gets to make one final offer. If they cannot reach an agreement in four days, both players get zero.

We assume Alan and David differ in their degree of impatience: Alan's discount factor is α per day and David's discount factor is β per day. We also assume that if a player is indifferent between two offers, he will accept the one that is most preferred by his opponent.

   (a) Draw the extensive form of this bargaining game.

   (b) Find the subgame perfect equilibrium of this bargaining game.

   (c) What is the final outcome of this game?

   (d) If David becomes more impatient what will happen to the equilibrium payoff to Alan?

B3. [15 Marks]

Two firms are competing in an oligopolistic industry. Firm 1, the larger of the two firms, is contemplating its capacity strategy, which could be either "aggressive" or "passive". The aggressive strategy involves a large increase in capacity aimed at increasing the firm's market share, while the passive strategy involves no change in the firm's capacity. Firm 2, the smaller competitor, is also pondering its capacity expansion strategy; it will also choose between an aggressive strategy and a passive strategy. The table below shows the profits associated with each pair of choices.

		Firm2	
		Aggressive	**Passive**
Firm 1	**Aggressive**	25, 9	33, 10
	Passive	30, 13	36, 12

    (a) If both firms decide their strategies simultaneously, what is the Nash equilibrium (or equilibria)? Is there any mixed strategy Nash equilibrium in this game? If yes, what is the mixed strategy Nash equilibrium (or equilibria)?

Now assume that Firm 1 can decide first and can credibly commit to its capacity expansion strategy.

    (b) Draw the extensive form of this sequential game. What is the subgame perfect equilibrium? What is the final outcome of this game?

    (c) If Firm 2 threatens to play "aggressive" if Firm 1 plays "aggressive", will it be credible to Firm 1? If this threat is not credible to Firm 1, what could Firm 2 do to make its threat credible?

B5. [15 Marks]

Suppose demand for a commodity is given by $y = 100 - p$. There are only two possible factories that can produce this commodity, each with cost function: $c_j = 50 + y_j^2$, where $j = 1, 2$ denotes the factory. The total market output is the sum of the outputs from these two plants.

(a) Find the efficient level of output and price for this market. Also, find the total profits of the two firms in this situation.

(b) Suppose the two firms form a cartel. Compute the profit maximizing total output, price, profits, and deadweight loss of the cartel in this situation.

(c) Instead of a cartel, suppose the two plants are owned by Cournot duopolists. Find the Cournot-Nash equilibrium output by each firm, the price, and the total profit. Also compute the deadweight loss associated with the Cournot duopoly.

(d) Instead of the Cournot assumption, suppose that firm 1 sets its output before firm 2 does. Firm 2 does observes the output choice of firm 1 before it makes its own output choice. Find the Stackelberg equilibrium output produced by each firm, the price, and total profit. Also compute the deadweight loss in this situation.

(e) Compare the results you found in (a), (b), (c), and (d).

B6. [15 Marks]

Two firms, Firm 1 and Firm 2, are competing in an oligopolistic industry. They produce an identical product. But Firm 1 does it at a lower cost than Firm 2. Firm 1 has a constant marginal cost of $15 and firm 2 has a constant marginal cost of $30. The market demand for the commodity is $p = 120 - y$, where y is aggregate output.

(a) Suppose that firms choose quantities. Find both best-response functions. Remember, marginal costs are different, so the best response functions will not be symmetric. Find the Cournot-Nash equilibrium quantities. Illustrate your results in a diagram.

(b) Suppose that the firms choose prices instead of quantities and that prices must be announced in dollars and cents. (That is, $15.71, and $39.00 are permissible prices, but $45.975 is not.) What are the Bertrand equilibrium prices? How much does each firm earn in the Bertrand equilibrium?