Technical Appendix for "Real Exchange Rate Fluctuations and the Dynamics of Retail Trade Industries on the U.S.-Canada Border"*

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The purpose of this appendix is threefold. First, we present a more detailed model than that in the text. The model incorporates U.S. and Canadian currencies as units of account and "sticky" producers that make both entry and nominal pricing decisions in advance of all shocks. This is the model we refer to in the discussion of industry dynamics with sticky prices in Section I of the text. Second, we present our procedure for imputing missing payroll observations from the County Business Patterns dataset. Third and finally, we present the GMM estimator we use, based on Blundell and Bond (1998).

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I An Expanded Model of Industry Dynamics with Cross-Border Shopping

In this section, we discuss an extension of the model in the text in which we include money as a unit of account, nominal disturbances, and sticky prices. Because our empirical results imply that net entry responds to real exchange rate fluctuations, we incorporate a role for entry to play in our model. To do so, we assume that there are two types of retailers, fast-entry flexible-price retailers and slow-entry sticky price retailers. We call these two types of retailers "flexible" and "sticky". As in the baseline model, there is free-entry by flexible retailers. In contrast, the number of potential sticky retailers is fixed. Sticky retailers have identical marginal cost structures and fixed costs that increase with the number of operating sticky retailers. Therefore, the number of sticky retailers operating in each country is also determined by free-entry.

To model the idea that some retailers are sticky while others are flexible, we consider the following three-stage game. First, all potential sticky-price retailers make their entry decisions and choose their nominal prices. Second, nominal shocks and real cost shocks are realized. Finally, flexible retailers enter and choose their nominal prices, and all consumers make their purchase decisions.

A Currencies and Consumers

As in the text, consumers in both countries have identical real incomes, ω, in units of the homogeneous good, which we henceforth refer to as wheat. The law of one price holds for wheat. There are two currencies, U.S. dollars and Canadian dollars, which are used only as units of account. The price of one unit of wheat in U.S. dollars is q_U, and the price of the same good in Canadian dollars is q_C. Because the law of one price holds for wheat, the nominal exchange rate is therefore, $e = q_U/q_C$. All retailers set prices in units of their home currencies. Let p_{iU} and p_{jC} denote the prices chosen by retailer i in county U and retailer j

in county C, and let x_{iU} and x_{jC} denote their respective quantities. Consumers preferences are as in the text, so the output demanded by consumers of retailer iU is

$$x_{iU} = \frac{(1-\lambda)\, S_U\,(\gamma q_U \omega)\, p_{iU}^{\frac{1}{\nu-1}}}{\int_0^{N_U} p_{\ell U}^{\nu/(\nu-1)} d\ell} + \frac{\lambda\gamma\,(S_U q_U \omega + S_C e q_C \omega)\, p_{iU}^{\frac{1}{\nu-1}}}{\int_0^{N_U} p_{\ell U}^{\nu/(\nu-1)} d\ell + \int_0^{N_C} (eP_{\ell C})^{\nu/(\nu-1)}\, d\ell},$$

where $q_U\omega$ and $q_C\omega$ are the nominal incomes of U.S. and Canadian consumers. A similar equation characterizes the demand for a typical Canadian retailer.

B Technology

All flexible retailers in each county share the same technology. In U, the fixed cost and marginal cost are F_{fU} and c_{fU}, both denominated in U.S. dollars. We denote their real counterparts with $\phi_{fU} = F_{fU}/q_U$ and $\kappa_{fU} = c_{fU}/q_U$. In Canada, these nominal and real quantities are denoted F_{fC}, c_{fC}, ϕ_{fC}, and κ_{fC}. The flexible-firm technology is widely available in both countries. In contrast, there is a limited supply of sticky retailers. If N_{sU} sticky retailers enter in U, then the sticky retailers' nominal fixed cost is $F_{s_U}\,(N_{sU}/S_U)^{\theta}$, where $\theta > 0$. All sticky retailers in U share the same nominal marginal cost, c_{sU}. The real counterparts to these costs are $\phi_{sU}\,(N_{sU}/S_U)^{\theta}$ and κ_{sU}. Again, the analogous Canadian quantities are denoted with the "C" subscript. Let N_{fU} and N_{fC} denote the number of flexible retailers in U and C, so that

$$N_U = N_{fU} + N_{sU},$$

$$N_C = N_{fC} + N_{sC}.$$

C Shocks

To incorporate aggregate uncertainty we assume that c_{sU}, c_{fU}, q_U, c_{sC}, c_{fC}, and q_C, are random variables, that flexible retailers make their entry decisions and choose their prices after their realizations, and that sticky retailers make their entry decisions and choose their prices before their realizations. We call a change in either q_U or q_C that leaves retailers' real marginal costs unchanged a *nominal* disturbance. To simplify the analysis, we assume that $\frac{c_{fU}}{c_{sU}} = \frac{c_{fC}}{c_{sC}} = \tau > 1$ *always*. This assumption implies that nominal disturbances do not alter the real (variable) cost benefits of being a sticky firm.

D Entry and Price Setting by Flexible Retailers

It is straightforward to show that flexible retailers' profit maximizing pricing decisions follow the familiar inverse-elasticity rule, as in the text. Therefore, we have the price choices of all flexible retailers within each county are identical and equal to

$$p_{fU} = c_{fU}/\nu,$$
$$p_{fC} = c_{fC}/\nu.$$

The free-entry condition for flexible retalers in U is therefore

(1)
$$\frac{(1-\lambda)\,S_U\,(\gamma q_U \omega)}{\int_0^{N_U} p_{\ell U}^{\nu/(\nu-1)} d\ell} + \frac{\lambda\gamma\,(S_U q_U \omega + S_C e q_C \omega)}{\int_0^{N_U} p_{\ell U}^{\nu/(\nu-1)} d\ell + \int_0^{N_U} (\ell P_{\ell C})^{\nu/(\nu-1)}\, d\ell} = \frac{\phi_{fU}}{(1-\nu)\left(\frac{c_{fU}}{\nu}\right)^{\frac{\nu}{1-\nu}}}$$

A similar free-entry condition hols for flexible retailers in C.

E Price Setting by Sticky Retailers

A sticky retailer must choose its nominal price before the realizations of any shocks. The profit-maximization problem of a sticky retailer in U is

(2)
$$\mathbf{E}\left[\pi_{sU}\right] = \max_{p_{sU}} \mathbf{E}\left[\left(\frac{(1-\lambda)\,S_U\,(\gamma q_U\omega)\,p_{sU}^{\frac{1}{\nu-1}}}{\int_0^{N_U} p_{\ell U}^{\nu/(\nu-1)} d\ell} + + \frac{\lambda\gamma\,(S_U q_U\omega + S_C e q_C \omega)\,p_{sU}^{\frac{1}{\nu-1}}}{\int_0^{N_U} p_{\ell U}^{\nu/(\nu-1)} d\ell + \int_0^{N_C} (e p_{\ell C})^{\nu/(\nu-1)}\, d\ell}\right)(p_{sU} - c_{sU})\right]$$
$$- \phi_{sU}\left(\frac{N_{sU}}{S_U}\right)^{\theta},$$

where the expectation is taken over the exogenous random variables c_{sU}, q_C, q_U, as well as the endogenous variables N_U, N_C, p_{fU} and p_{fC}. The prices of Canadian sticky retailers, p_{sC}, and the fixed cost (and therefore the number of sticky retailers) are presumed to be known when choosing p_{sU}. The presence of flexible retailers considerably simplifies this problem, because the free-entry condition (1) must hold for every realization of the exogenous random variables. Substitution of (1) into (2) within the expectations operator yields

$$\mathbf{E}\left[\pi_{sU}\right] = \max_{p_{sU}} \mathbf{E}\left[\frac{\phi_{fU}}{1-\nu}\left(\frac{c_{fU}}{\nu}\right)^{\frac{\nu}{1-\nu}} p_{sU}^{\frac{1}{\nu-1}}\left(p_{sU}-c_{sU}\right)\right] - \phi_{sU}\left(\frac{N_{sU}}{S_U}\right)^{\theta}.$$

Imposing the assumption that $c_{fU}/c_{sU} = \tau$ always, we can easily show that

$$p_{sU} = \frac{\mathbf{E}\left[c_{sU}\right]}{\nu}\left(\frac{\mathbf{E}\left[c_{sU}^{\frac{1}{1-\nu}}\right]}{\mathbf{E}\left[c_{sU}\right]\mathbf{E}\left[c_{sU}^{\nu/1-\nu}\right]}\right). \tag{3}$$

A similar expression holds for p_{sC}. The first term on the right-hand side of (3) is the optimal price choice in sticky-price models with no entry by flexible producers. To better understand the term in brackets, note that the Cauchy-Schwartz inequality implies that for any constant ζ such that $\mathbf{E}\left[(\zeta c_{sU})^{\frac{1}{1-\nu}}\right] > 1$, then

$$\mathbf{E}\left[(\zeta c_{sU})^{\frac{1}{1-\nu}}\right] = \mathbf{E}\left[\zeta c_{sU}\left(\zeta c_{sU}\right)^{\nu/(1-\nu)}\right] \leq \mathbf{E}\left(\zeta c_{sU}\right)^{\frac{1}{2}} \mathbf{E}\left[(\zeta c_{sU})^{\frac{\nu}{1-\nu}}\right]^{\frac{1}{2}}$$

We can square the right hand side and divide the result by $\zeta^{1/(1-\nu)}$. to get

$$\mathbf{E}\left[c_{sU}^{\frac{1}{1-\nu}}\right] < \mathbf{E}\left[c_{sU}\right]\mathbf{E}\left[c_{sU}^{\nu/1-\nu}\right].$$

While unessential for our main point, this says that sticky retailers lower their markups below what they would be if there were no entry by flexible retailers. That is, they drop their preset price so that they do not lose sales during contractions to flexible producers. Also, the presence or absence of cross-border shoppers has no impact on sticky retailers' pricing decisions. This will not generally be the case without the presence of flexible retailers.

F Free Entry by Sticky Retailers

To determine the number of sticky retailers in each county, we impose the free-entry condition that the profits of the marginal entrant in each county equals zero. The simplification of a sticky retailer's profit maximization problem using flexible retailers' free-entry condition aids in this enterprise as well. If we calculate a sticky retailer's expected profits and choose the number of such retailers in each county to set profits equal to zero, we get

$$N_{sU} = S_U \left(\frac{\phi_{fU}}{\phi_{sU}} \right)^{\frac{1}{\theta}} \left[\tau^{\nu/(1-\nu)} \mathbf{E} \left[c_{sU}^{\frac{1}{1-\nu}} \right]^{\frac{\nu}{\nu-1}} \mathbf{E} \left[c_{sU}^{\frac{\nu}{1-\nu}} \right]^{\frac{1}{1-\nu}} \right]^{\frac{1}{\theta}}.$$

A similar expression holds for N_{sC}. By assumption, N_{sU} and N_{sC} do not depend on any of the shocks we consider.

G Free Entry by Flexible Retailers

We can now turn to the determination of N_{fU} and N_{fC}. The two free-entry conditions that these must simultaneously satisfy are

(4)

$$\frac{\phi_{fU}}{(1-\nu)} \left(\frac{c_{fU}}{\nu} \right)^{\frac{\nu}{1-\nu}} = \frac{(1-\lambda)\,\gamma\omega S_U}{N_{sU} p_{sU}^{\nu/(\nu-1)} + N_{fU} p_{fU}^{\nu/(\nu-1)}} + \frac{\lambda\gamma\omega\,(S_U + S_C)}{N_{sU} p_{sU}^{\nu/(\nu-1)} + N_{fU} p_{fU}^{\nu/(\nu-1)} + N_{sC}\,(e p_{sC})^{\nu/\nu-1} + N_{fC}\,(e p_{fC})^{\nu/(\nu-1)}}$$

(5)

$$\frac{\phi_{fC}}{(1-\nu)} \left(\frac{c_{fC}}{\nu} \right)^{\frac{\nu}{1-\nu}} = \frac{(1-\lambda)\,\gamma\omega S_C}{N_{sC} p_{sC}^{\nu/(\nu-1)} + N_{fC} p_{fC}^{\nu/(\nu-1)}} + \frac{\lambda\gamma\omega\,(S_U + S_C)}{N_{sU}\,(e^{-1} p_{sU})^{\nu/\nu-1} + N_{fU}\,(e^{-1} p_{fU})^{\nu/(\nu-1)}\, N_{sC} p_{sC}^{\nu/(\nu-1)} + N_{fC} p_{fC}^{\nu/(\nu-1)}}$$

G.1 The Solution at Parity

In general, these equations do not admit a closed-form solution. However, they do admit such a solution in the case of "parity", when $\phi_{fC} = \phi_{fU}$ and $c_{fC/q_C} = c_{fU/q_U}$. In this case, we can divide the second equation by $e^{\frac{\nu}{\nu-1}}$, subtract (5) from (4) and rearrange to get

$$N_{sC}\left(eP_{sC}\right)^{\nu/(\nu-1)}+N_{fC}\left(eP_{fC}\right)^{\nu/(\nu-1)}=\frac{S_C}{S_U}\left(N_{sU}P_{sU}^{\nu/(\nu-1)}+N_{fU}P_{fU}^{\nu/(\nu-1)}\right)$$

Substituting this into (4) yields

$$\frac{(1-\lambda)\,\gamma\omega S_U}{N_{sU}P_{sU}^{\nu/(\nu-1)}+N_{fU}P_{fU}^{\nu/(\nu-1)}}+\frac{S_U\left(1+\frac{s_C}{S_U}\right)}{\left(1+\frac{S_C}{S_U}\right)\left(N_{sU}P_{sU}^{\nu/(\nu-1)}+N_{fU}P_{fU}^{\nu/(\nu-1)}\right)}$$
$$=\frac{\phi_{fU}}{(1-\nu)}\left(\frac{c_{fU}}{\nu}\right)^{\frac{\nu}{1-\nu}}$$

Which obviously simplifies to

$$\frac{\gamma\omega S_U}{N_{sU}P_{sU}^{\nu/(\nu-1)}+N_{fU}P_{fU}^{\nu/(\nu-1)}}=\frac{\phi_{fU}}{(1-\nu)}\left(\frac{c_{fU}}{\nu}\right)^{\nu/(1-\nu)},$$

the same free entry condition as in the model without cross-border shopping. The analogous derivation holds for the Canadian side of the border as well. Therefore,

$$N_{fU}=S_U\left(\frac{\gamma\omega\,(1-\nu)}{\phi_{fU}}-\left(\frac{c_{fU}}{\nu}\right)^{\nu/(1-\nu)}\left[\tau_U^{\nu/(1-\nu)}\mathbf{E}\left[c_{sU}^{\frac{1}{1-\nu}}\right]^{\frac{\nu}{\nu-1}}\mathbf{E}\left[c_{sU}^{\frac{\nu}{1-\nu}}\right]^{\frac{1}{1-\nu}}\right]^{\frac{1}{\theta}}\right)$$
$$\times\left(\frac{\mathbf{E}\left[c_{sU}\right]}{\nu}\left(\frac{\mathbf{E}\left[c_{sU}^{\frac{1}{1-\nu}}\right]}{\mathbf{E}\left[c_{sU}\right]\mathbf{E}\left[c_{sU}^{\frac{\nu}{1-\nu}}\right]}\right)^{\nu/(\nu-1)}\right)$$

in the case of parity. An analogous expression also holds for N_{fC}. We denote these parity solutions with $\overline{N}_{fU}$ and $\overline{N}_{fC}$, and we presume that the parameter values are such that $\overline{N}_{fU}$ and $\overline{N}_{fC}$ are both strictly positive.

G.2 The Log-Linear Solution for the General Case

We wish to consider the industry's responses to nominal and real shocks. To do so, we take a log-linear approximation of the two free-entry conditions for flexible retailers around their solution at parity. We assume that $\kappa_{fU}=\kappa_{fC},\kappa_{sU}=\kappa_{sC}$, and that the distributions of c_{sU} and c_{sC} are identical up to scale so that sticky retailers make identical price choices and

$\frac{N_{fU}}{N_{fC}} = \frac{N_{sU}}{N_{sC}} = \frac{S_U}{S_C}$ and $\frac{N_{fa}}{N_{sU}} = \frac{N_{fC}}{N_{sC}} = \psi$ at parity. To simplify the free-entry conditions, we define

$$d_C = p_{sC}/p_{fC},$$

$$d_U = p_{sU}/p_{fU},$$

$$r = \frac{e^{-1}p_{fU}}{p_{fC}}.$$

That is, d_C and d_U are the discounts offered by sticky retailers (relative to their domestic flexible counterparts) in C and U, while r is the real exchange rate formed using flexible retailers' prices. Under the model's assumptions, only nominal disturbances impact d_C and d_U, while only real disturbances that change flexible retailers' marginal costs impact r.

With these definitions in hand, we can manipulate the free-entry conditions (4) and (5) to get

$$\frac{\phi_{fU}}{1-\nu} = \frac{S_U}{N_{fU}}\left[\frac{(1-\lambda)\,\gamma\omega}{\frac{N_{sU}}{N_{fU}}d_U^{\nu/(\nu-1)}+1} + \frac{\lambda\gamma\omega\,(1+S_C/S_U)}{\frac{N_{sU}}{N_{fU}}d_U^{\nu/(\nu-1)}+1+\frac{N_{sC}}{N_{fU}}\,(d_C/r)^{\nu/(\nu-1)}+\frac{N_{fC}}{N_{fU}}r^{\nu/(\nu-1)}}\right]$$

$$\frac{\phi_{fC}}{1-\nu} = \frac{S_C}{N_{fC}}\left[\frac{(1-\lambda)\,\gamma\omega}{\frac{N_{sC}}{N_{fC}}d_C^{\nu/(\nu-1)}+1} + \frac{\lambda\gamma\omega\,(1+S_U/S_C)}{\frac{N_{sU}}{N_{fC}}\,(d_U)^{\nu/(\nu-1)}+\frac{N_{fU}}{N_{fC}}r^{\nu/(\nu-1)}+\frac{N_{sC}}{N_{fC}}d_C^{\nu/(\nu-1)}+1}\right]$$

The log-linear approximation to these equations around the point of parity is

$$\eta\cdot\begin{bmatrix}\ln\left(N_{fU}/\overline{N}_{fU}\right)\\ \ln\left(N_{fC}/\overline{N}_{fC}\right)\end{bmatrix} + \delta\begin{bmatrix}\ln r\\ \ln\left(d_U/d\right)\\ \ln\left(d_C/d\right)\end{bmatrix} = 0,$$

where d is the common point where we evaluate d_U and d_C. The elements of η are

$$\eta_{11} = \frac{1}{1+\psi d^{\nu/(\nu-1)}}\left(-1+\lambda\frac{S_C}{S_U+S_C}\right),$$

$$\eta_{12} = \frac{1}{1+\psi d^{\nu/(\nu-1)}}\frac{-\lambda S_C}{S_U+S_C},$$

$$\eta_{21} = \frac{1}{1+\psi d^{\nu/(\nu-1)}}\frac{\lambda S_U}{S_C+S_U},$$

$$\eta_{22} = \frac{1}{1+\psi d^{\nu/(\nu-1)}}\left(-1+\frac{\lambda S_U}{S_U+S_C}\right);$$

and the elements of δ are

$$\begin{aligned}
\delta_{11} &= \frac{1}{1+\psi d^{\nu/(\nu-1)}}\left(-\lambda\frac{S_C}{S_U+S_C}\cdot\frac{\nu}{1-\nu}\right),\\
\delta_{21} &= \frac{1}{1+\psi d^{\nu/(\nu-1)}}\left(\frac{\lambda S_U}{S_U+S_C}\cdot\frac{\nu}{1-\nu}\right),\\
\delta_{12} &= \frac{\psi d^{\nu/(\nu-1)}}{1+\psi d^{\nu/(\nu-1)}}\cdot\frac{\nu}{1-\nu}\left(1-\frac{\lambda S_C}{S_U+S_C}\right),\\
\delta_{22} &= \frac{\psi d^{\nu/(\nu-1)}}{1+\psi d^{\nu/(\nu-1)}}\cdot\frac{\nu}{1-\nu}\frac{\lambda S_C}{S_U+S_C},\\
\delta_{31} &= \frac{\psi d^{\nu/(\nu-1)}}{1+\psi d^{\nu/(\nu-1)}}\cdot\frac{\nu}{1-\nu}\frac{\lambda S_C}{S_U+S_C},\\
\delta_{32} &= \frac{\psi d^{\nu/(\nu-1)}}{1+\psi d^{\nu/(\nu-1)}}\cdot\frac{\nu}{1-\nu}\left(1-\frac{\lambda S_U}{S_U+S_C}\right).
\end{aligned}$$

If we multiply both η and δ by $1+\psi d^{\nu/(\nu-1)}$ we get

$$\begin{aligned}
\begin{bmatrix} \ln\left(N_{fU}/\overline{N}_{fU}\right) \\ \ln\left(N_{fC}/\overline{N}_{fC}\right) \end{bmatrix} &= \begin{bmatrix} \frac{-1+\lambda S_C}{S_U+S_C} & \frac{-\lambda S_C}{S_U+S_C} \\ \frac{-\lambda S_U}{S_U+S_C} & \frac{-1+\lambda S_U}{S_U+S_C} \end{bmatrix}^{-1} \\
&\times \frac{\nu}{1-\nu}\begin{bmatrix} \frac{-\lambda S_C}{S_U+S_C} & \psi d^{\nu/(\nu-1)}\left(\frac{-1+\lambda S_C}{S_U+S_C}\right) & \frac{\lambda S_C}{S_U+S_C}\psi d^{\nu/(\nu-1)} \\ \frac{\lambda S_U}{S_U+S_C} & \frac{\lambda S_U}{S_U+S_C}\psi d^{\nu/(\nu-1)} & \psi d^{\nu/(\nu-1)}\left(1-\frac{\lambda S_U}{S_U+S_C}\right) \end{bmatrix} \\
&\times \begin{bmatrix} \ln r \\ \ln\left(d_U/d\right) \\ \ln\left(d_C/d\right) \end{bmatrix}
\end{aligned}$$

The matrix inverse is

$$\begin{bmatrix} \frac{-1+\lambda S_C}{S_U+S_C} & \frac{-\lambda S_C}{S_U+S_C} \\ \frac{-\lambda S_U}{S_U+S_C} & \frac{-1+\lambda S_U}{S_U+S_C} \end{bmatrix}^{-1} = \frac{1}{1-\lambda}\begin{bmatrix} -1+\frac{\lambda S_U}{S_U+S_C} & \frac{\lambda S_C}{S_U+S_C} \\ \frac{\lambda S_U}{S_C+S_C} & -1+\frac{\lambda S_C}{S_U+S_C} \end{bmatrix}$$

Plugging this in and solving, we get that

$$\text{(6)}\quad \begin{bmatrix} \ln\left(N_{fU}/\overline{N}_{fU}\right) \\ \ln\left(N_{fC}/\overline{N_{fC}}\right) \end{bmatrix} = \frac{\nu}{1-\nu}\begin{bmatrix} \frac{\lambda}{1-\lambda}\frac{S_C}{S_U+S_C} & -\psi d^{\nu(\nu-1)} & 0 \\ \frac{\lambda}{1-\lambda}\frac{S_C}{S_U+S_C} & 0 & -\psi d^{\nu(\nu-1)} \end{bmatrix} \times \begin{bmatrix} \ln r \\ \ln\left(d_U/d\right) \\ \ln\left(d_C/d\right) \end{bmatrix}$$

Equation (6) is the log-linear solution to the model.

H Discussion of the Solution

There are three features of this solution worth noting. First, real disturbances that change r impact N_{fU} and N_{fC} in exactly the same way as they do in the simpler model in the text. Because N_{sU} and N_{sC} do not respond to r by construction, we can use this result to show that

$$\frac{\partial \ln N_U}{\partial \ln r} = \frac{\overline{N}_{fU}}{N_{sU} + \overline{N}_{fU}} \frac{\nu}{1-\nu} \frac{\lambda}{1-\lambda} \frac{S_C}{S_U + S_C}. \tag{7}$$

Under the assumptions made in the text, each retailer's payroll is a constant fraction of its marginal cost. A real disturbance that changes r leaves the average payroll of both flexible and sticky retailers unchanged. However, it can impact overall average payroll by changing the composition of establishments between these two groups. If we let W_{fU} and W_{sU} denote the average payrolls of flexible and sticky retailers, then we get that

$$\frac{\partial \ln W_U}{\partial \ln r} = \left(\frac{W_{fU}\overline{N}_{fU}}{W_{sU}N_{sU} + W_{fU}\overline{N}_{fU}} - \frac{\overline{N}_{fU}}{N_{sU} + \overline{N}_{fU}} \right) \frac{\nu}{1-\nu} \frac{\lambda}{1-\lambda} \frac{S_C}{S_U + S_C} \tag{8}$$

Equations (7) and (8) imply that the empirical model's specification of the sensitivity measure with respect to real exchange-rate fluctuations driven by real shocks is correct in this more complicated environment.

The second important feature of (6) is that the impact of nominal disturbances that erode or inflate sticky retailers' preset nominal prices in U (and so change d_U) does not depend on the presence or absence of cross-border shopping. Because such nominal disturbances can be expected to impact both border and interior counties, their effects will then be absorbed into the coefficients on the time-dummies in our empirical model. Third and last, Canadian nominal disturbances that change d_C have no impact on N_{fU}. Together, these two features of the solution imply that purely nominal disturbances that change the real prices of sticky retailers have no particular impact on border communities retail trade industries in the presence of flexible retailers whose entry and pricing decisions can respond to these shocks. Therefore, such a framework is not capable of reconciling our empirical results with an important role for retail-level price stickiness.

II Data Imputation Procedure

This section describes our procedure for replacing payroll data for retail trade industries in the County Business Patterns data set that has been withheld by the Census to preserve confidentiality. The basic idea is to use the information that we do have on establishment counts by size class at the county level and total payroll at the state level to estimate the relationship between the number of establishments and total payroll among those counties where the data has been withheld. Fitted values from this estimated regression then serve as estimates of the withheld payroll data.

To begin with, focus on a particular retail trade industry during a particular year. Let W_c^s denote the total payroll in that industry in county c of state s, and let W^s denote the statewide payroll in that industry for state s. If $C(s)$ is the set of all counties in state s, then

$$W^s = \sum_{c \in C(s)} W_c^s.$$

We assume that observations of W^s are available for every state. Because the number of retail establishments in a given state is usually large, data suppression is typically not a problem at the state level in this data set. On the other hand, suppression of observations of W_c^s for individual counties is common. What is always reported for each county is the number of establishments belonging to several predetermined size classes (based on mid-March employment). Let J denote the set of such size classes and $N_c^s(j)$ denote the number of establishments in class j in county c of state s. The data replacement procedure is based on a regression model of W_c^s on $N_c^s(j)$ restricted to those counties where the census has withheld publication of W_c^s. Let $\mathfrak{W}^s$ denote the set of all counties in state s for which the Census has withheld publication of W_c^s. Then the basic regression model is

$$W_c^s = \sum_{j \in J} \beta_j N_c^s(j) + u_c^s \quad (9)$$

$$\mathbf{E}[u_c^s] = 0$$

for all $c \in \mathfrak{W}^s$. The coefficients β_j are constant across both counties and states. That is, the regression equation specifies that the total payroll in a county equals a linear function of the number of establishments in each size class plus a mean zero error term.

The obvious impediment to estimating the equation is that the dependent variable is withheld for all of the observations of interest. To overcome this, we can aggregate the equation to the state level, where the aggregated dependent variable is observable. To do so, define $\widetilde{W}^s$ as the payroll in all counties in state s for which payroll data is withheld. This can be constructed as statewide payroll minus payroll at all counties at which payroll was reported. That is

$$\widetilde{W}^s = W^s - \sum_{c \in \overline{\mathfrak{W}}^s} W_c^s,$$

where $\overline{\mathfrak{W}}^s$ is the complement of $\mathfrak{W}^s$. If we then define

$$\widetilde{N}^s (j) = \sum_{c = \mathfrak{W}^s} N_c^s (j)$$

then aggregating (9) for state s yields

$$\widetilde{W}^s = \sum_{j \in J} \beta_j \widetilde{N}^s (j) + \widetilde{u}^s, \tag{10}$$

where

$$\widetilde{u}^s = \sum_{c = W^s} u_c^s.$$

If we calculate the dependent variables and regressors for (10) for each state, then the coefficients β_j can be estimated by applying the regression to the state level data. The fitted values of this estimated model can then be used to construct estimates of the withheld county-level payroll data. When implementing this procedure, we construct separate estimates of β_j for each year and industry in our sample.

III GMM Estimation

In this section, we consider the GMM estimation procedure described in the text in more detail. Several aspects of the paper's empirical model generalize quite readily. Here we consider the appropriately generalized version with m dependent variables, an autoregressive order of p, and k current and lagged values of the real exchange rate. The resulting estimating equation is

$$\underset{(m\times 1)}{y_{it}} = \underset{(m\times 1)}{\alpha_i} + \underset{(m\times 1)}{\mu_t} + \sum_{l=1}^{p} \underset{(m\times m)}{\Lambda_l} \underset{(m\times 1)}{y_{it-l}} + \underset{(m\times k)}{\beta'} \left(s_i \times \underset{(k\times 1)}{e_t} \right) + \underset{(m\times 1)}{\varepsilon_{it}} . \tag{11}$$

The dimensions of all vectors and matrices appear below them. The vector y_{it} contains the period t values of the m variables describing a particular retail trade industry in county i. In the baseline model, m equals 2 and these variables are the logarithms of total establishments and their average payroll. The vector α_i is the county specific intercept term, and the vector μ_t is an aggregate disturbance that impacts all counties' industries in period t. The matrices Λ_l contain the model's autoregressive coefficients. The scalar s_i is the index of county i's sensitivity to real exchange rates described in the text, and the vector e_t contains the k current and lagged realizations of the real exchange rate. For most counties in our sample, $s_i = 0$. Finally, the matrix β contains the elasticities of y_{it} with respect to a change in e_t for a county with s_i equal to one, and ε_{it} is a disturbance vector.

We observe a balanced panel of the variables in y_{it} for T periods and N counties. Equation (11) only describes the evolution of y_{it} for t between p and T. The initial p realizations of y_{it} will play a key role in estimation of the unknown parameters in (11). We make the following assumptions on the model's error terms and parameters.

1. The roots of $|I - \sum_{l=1}^{p} \Lambda_l L^p|$ all lie strictly outside of the unit circle.

2. $\Pr[s_i = 0] > 0$.

3. The individual specific intercept α_i and the error terms ε_{it}, $t = p+1, \ldots, T$ are independently distributed across individuals and

(a) $\mathbf{E}[\alpha_i | s_i = 0] = 0$,

(b) $\mathbf{E}[\varepsilon_{it}] = 0$, $t = p+1, \ldots, T$,

(c) $\mathbf{E}[\varepsilon_{it}\varepsilon'_{i\tau} | s_i = 0] = 0$, if $t \neq \tau$,

(d) $\mathbf{E}[\alpha_i \varepsilon'_{it}] = 0$, $t = p+1, \ldots, T$,

(e) $\mathbf{E}[s_i \varepsilon_{it}] = 0$, $t = p+1, \ldots, T$,

(f) $\mathbf{E}[\alpha_i \alpha'_i] < \infty$

(g) $\mathbf{E}[\varepsilon_{it}\varepsilon'_{it}] < \infty$, $t = p+1, \ldots, T$.

4. If $s_i = 0$, then the first p values of y_{it} satisfy

$$y_{it} = \mu_t + \left(I - \sum_{l=1}^{p} \Lambda_l \right)^{-1} \alpha_i + u_{it}, \;\; t = 1, \ldots, p, \tag{12}$$

where

(a) $\mathbf{E}[u_{it}] = 0$, $t = 1, \ldots, p$,

(b) $\mathbf{E}[\alpha_i u'_{it}] = 0$, $t = 1, \ldots, p$,

(c) $\mathbf{E}[s_i u_{it}] = 0$, $t = 1, \ldots, p$,

(d) $\mathbf{E}[u_{i\tau}\varepsilon'_{it}] = 0$, for all $\tau = 1, \ldots, p$ and $t = p+1, \ldots, T$,

(e) $\mathbf{E}[u_{it} u'_{i\tau}] < \infty$ for all $\tau = 1, \ldots, p$ and $t = 1, \ldots, p$.

5. The regressors $e_{p+1}, e_{p+2}, \ldots, e_T$ are known constants.

Assumption 1 implies that the autoregressive system in (11) is stable, and Assumption 2 asserts that cross-border shopping does not impact a positive fraction of our sample counties. This is clearly the case in our sample. Given the presence of the time effects in (11), 3(a) and 3(b) are normalizations. Assumption 3(c) restricts the error term in (11) to be uncorrelated through time. Assumptions 3(d) and 3(e) assert that ε_{it} cannot be forecasted using a linear function of α_i and s_i. In the case where $s_i = 0$, Assumptions 4(a) and 4(b) assert that the deviations of y_{i1} through y_{ip} from their unconditional means are uncorrelated with those means. Assumption 4(d) asserts that ε_t cannot be forecasted using linear functions of $u_{i\tau}$. Notice that assumptions 3(d) and 4(b) do not restrict the higher moments of ε_{it} or u_{it} from being dependent on α_i, so the model allows for general forms of heteroskedasticity. Also note that we do not constrain the covariance between α_i and s_i to equal zero. The remaining assumptions are regularity conditions that guarantee existence of second moments for y_{it}.

A Moment Conditions

To estimate the unknown parameters in (11), we derive moment conditions which are functions of the observed data that are satisfied only at the true parameter values. We then use these moment conditions in a GMM estimation procedure to produce consistent parameter estimates. Our derivation of the moment conditions closely follows Blundell and Bond (1998). The differences between our derivation and theirs is minor, and only allow for the inclusion of the independent variables $s_i e_t$ and a vector (as opposed to univariate) autoregression. Our distributional theory for the estimator is the same as Blundell and Bond's, letting N go to infinity while T is held fixed.

If $s_i = 0$, we can use (11) and (12) to write that

$$y_{ip+1} = \widetilde{\alpha}_i + \mu_{p+1} + \sum_{l=1}^{p} \Lambda_l \left(\widetilde{\alpha}_i + u_{ip+1-l}\right) + \varepsilon_{ip+1-l}.$$

In general, for $t \geq p+1$, we get

$$y_{it} = \widetilde{\alpha}_i + \sum_{j=0}^{t-p-1} \psi_j \varepsilon_{it-j} + \sum_{j=t-p}^{t-1} \psi_j u_{it-j} + \sum_{j=0}^{t-1} \psi_j \mu_{t-j}, \tag{13}$$

where ψ_j is defined recursively with

$$\psi_0 = I_m$$

$$\psi_j = 0,\ \forall j < 0$$

$$\psi_j = \sum_{l=1}^{p} \Lambda_l \psi_{j-l},\ \forall j > 0.$$

Equation (13) and assumptions 3(c) and 4(d) imply that

$$\mathbf{E}\left[I\left\{s_i=0\right\}\Delta\varepsilon_{it}\cdot y'_{it-\tau}\right]=0,\ \forall t\geq p+2,\ 2\leq\tau\leq t-1, \tag{14}$$

where $I\left\{s_i=0\right\}$ is an indicator function that equals one if and only if $s_i=0$. Furthermore, (13) and assumptions 3(a), 3(b), 3(c), 3(d), 4(a), 4(b), and 4(d) imply that

$$\mathbf{E}\left[I\left\{s_i=0\right\}\times\left(\alpha_i+\varepsilon_{it}\right)\cdot\Delta y'_{it-\tau}\right]=0,\ \forall t\geq p+1,\ \tau\geq 1 \tag{15}$$

Note that many of the additional moment conditions implied by (15) are redundant. If we define $\widetilde{t}\,(t)=\max\left\{t+1,p+1\right\}$ and we choose a $t^\star>\widetilde{t}\,(t)$, then we can write that

$$\begin{aligned}\mathbf{E}\left[I\left\{s_i=0\right\}\times\left(\alpha_i+\varepsilon_{it^\star}\right)\cdot\Delta y'_{it}\right]&=\mathbf{E}\left[I\left\{s_i=0\right\}\left(\alpha_i+\varepsilon_{i\widetilde{t}(t)}+\sum_{\tau=\widetilde{t}(t)+1}^{t^\star}\Delta\varepsilon_{i\tau}\right)\cdot\Delta y'_{it}\right]\\&=\mathbf{E}\left[I\left\{s_i=0\right\}\left(\alpha_i+\varepsilon_{i\widetilde{t}(t)}\right)\cdot\Delta y'_{it}\right]\\&+\sum_{\tau=\widetilde{t}(t)+1}^{t^\star}\mathbf{E}\left[I\left\{s_i=0\right\}\Delta\varepsilon_{i\tau}\cdot y'_{it}\right]-\sum_{\tau=\widetilde{t}(t)+1}^{t^\star}\mathbf{E}\left[I\left\{s_i=0\right\}\Delta\varepsilon_{i\tau}\cdot y'_{it-1}\right].\end{aligned}$$

Therefore, imposing

$$\mathbf{E}\left[I\left\{s_i=0\right\}\left(\alpha_i+\varepsilon_{i\widetilde{t}(t)}\right)\cdot\Delta y'_{it}\right]=0,\ t=2,\ldots,T-1 \tag{16}$$

and (14) suffices to impose the entire set of moment conditions implied by (14) and (15).

Finally, 3(a) and 3(b) imply that

$$\mathbf{E}\left[I\left\{s_i=0\right\}\left(\alpha_i+\varepsilon_{it}\right)\right]=0, \tag{17}$$

and 3(e) implies that

$$\mathbf{E}\left[\Delta\varepsilon_{it}\cdot s_i\right]=0,\ \forall t\geq p+2. \tag{18}$$

Our full set of moment conditions used for parameter estimation is given by (14), (16), (17), and (18).

B Parameter Estimation

Let

$$\gamma = \left(\operatorname{vec}\left(\Lambda_1'\right)', \operatorname{vec}\left(\Lambda_2'\right)', \ldots, \operatorname{vec}\left(\Lambda_p'\right)', \operatorname{vec}\left(\beta'\right)', \mu_{p+1}', \mu_{p+2}', \ldots, \mu_T'\right)'$$

denote the vector of parameters of interest, and define

$$u_{it}\left(\gamma\right) = y_{it} - \mu_t + \sum_{l=1}^{p} \Lambda_l y_{it-l} + \beta'\left(s_i \times e_t\right),$$

for $t \geq p+1$. Let γ_0 denote the true parameter values. Then the moment conditions (14), (16), (17), and (18) can be rewritten as

$$\begin{aligned}
\mathbf{E}\left[\{s_i = 0\}\,\Delta u_{it}\left(\gamma_0\right) \cdot y_{it-\tau}'\right] &= 0,\ t = p+2, \ldots, T,\ 2 \leq \tau \leq t-1 \\
\mathbf{E}\left[I\left\{s_i = 0\right\} \times u_{i\tilde{t}(t)}\left(\gamma_0\right) \cdot \Delta y_{it}'\right] &= 0,\ t = 2, \ldots, T-1. \\
\mathbf{E}\left[I\left\{s_i = 0\right\} u_{it}\left(\gamma_0\right)\right] &= 0,\ \ t = p+1, \ldots, T \\
\mathbf{E}\left[\Delta u_{it} \cdot s_i\right] &= 0,\ t = p+2, \ldots, T.
\end{aligned}$$

To express these moment conditions in matrix form for a given individual, define the error vector $u_i\left(\gamma\right)$ to be

$$u_i\left(\gamma\right) = \begin{bmatrix} \Delta u_{ip+2}\left(\gamma\right) \\ \Delta u_{ip+3}\left(\gamma\right) \\ \vdots \\ \Delta u_{iT}\left(\gamma\right) \\ u_{ip+1}\left(\gamma\right) \\ u_{ip+2}\left(\gamma\right) \\ \vdots \\ u_{iT}\left(\gamma\right) \end{bmatrix},$$

and define the instrument vector z_i to be

$$z_i = \begin{bmatrix} I\{s_i = 0\} \\ s_i \\ I\{s_i = 0\} \cdot y_{i1} \\ I\{s_i = 0\} \cdot y_{i2} \\ \vdots \\ I\{s_i = 0\} \cdot y_{iT-2} \\ I\{s_i = 0\} \cdot \Delta y_{i2} \\ I\{s_i = 0\} \cdot \Delta y_{i3} \\ \vdots \\ I\{s_i = 0\} \cdot \Delta y_{iT-1} \end{bmatrix}.$$

Finally, define the moment selector matrix C to be a sparse matrix with row dimension equal to the number of valid moment conditions and a single element in the j'th column equal to one if

$$\mathbf{E}\left[\left(z_i \otimes u_i\left(\gamma_0\right)\right)_j\right] = 0$$

where the subscript j refers to the j'th element of that vector.

Let A_N be a square, positive definite matrix that has dimensionality equal to the row dimension of C. This matrix may be data dependent. Define the sample moment function $g_N(\gamma)$ as

$$g_N(\gamma) = C \cdot \frac{1}{N} \sum_{i=1}^{N} z_i \otimes u_i(\gamma).$$

Then the GMM estimator is the value of γ that minimizes

$$J_N(\gamma) = g_N(\gamma)' \cdot A_N \cdot g_N(\gamma).$$

To characterize the solution to this minimization problem, we can apply the rule for differentiating a quadratic form to get the first-order necessary condition which the GMM estimator, $\widehat{\gamma}_N$, must satisfy

$$\frac{\partial J_N\left(\widehat{\gamma}_N\right)}{\partial \gamma'} = 2 g_N\left(\widehat{\gamma}_N\right)' A_N \frac{\partial g_N\left(\widehat{\gamma}_N\right)}{\partial \gamma'} = 0.$$

To find a closed-form solution for $\widehat{\gamma}_N$, it is helpful to define

$$Y_i = \begin{bmatrix} \Delta y_{ip+2} \\ \Delta y_{ip+3} \\ \vdots \\ \Delta y_{iT} \\ y_{ip+1} \\ y_{ip+2} \\ \vdots \\ y_{iT} \end{bmatrix},$$

and

$$
X_i = \begin{bmatrix}
I_m \otimes \Delta y'_{ip+1} & I_m \otimes \Delta y'_{ip} & \cdots & I_m \otimes \Delta y'_{i2} & I_m \otimes s_i \Delta e'_{p+2} & -I_m & I_m & 0 & \cdots & 0 & 0 \\
I_m \otimes \Delta y'_{ip+2} & I_m \otimes \Delta y'_{ip+1} & \cdots & I_m \otimes \Delta y'_{i3} & I_m \otimes s_i \Delta e'_{p+3} & 0 & -I_m & I_m & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
I_m \otimes \Delta y'_{iT-1} & I_m \otimes \Delta y'_{iT-2} & \cdots & I_m \otimes \Delta y'_{iT-p} & I_m \otimes s_i \Delta e'_{T} & 0 & 0 & 0 & \cdots & -I_m & I_m \\
I_m \otimes y'_{ip} & I_m \otimes y'_{ip-1} & \cdots & I_m \otimes y'_{i1} & I_m \otimes s_i e'_{p+1} & I_m & 0 & 0 & \cdots & 0 & 0 \\
I_m \otimes y'_{ip+1} & I_m \otimes y'_{ip} & \cdots & I_m \otimes y'_{i2} & I_m \otimes s_i e'_{p+2} & 0 & I_m & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
I_m \otimes y'_{iT-1} & I_m \otimes y'_{iT-2} & \cdots & I_m \otimes y'_{iT-p} & I_m \otimes s_i e'_{T} & 0 & 0 & 0 & \cdots & 0 & I_m
\end{bmatrix}.
$$

Then we can write that

$$u_i(\gamma) = Y_i - X_i \gamma,$$

and that

$$
\begin{aligned}
C(z_i \otimes u_i(\gamma)) &= C(z_i \otimes (Y_i - X_i\gamma)) \\
&= C(z_i \otimes Y_i) - C(z_i \otimes X_i \gamma) \\
&= C(z_i \otimes Y_i) - C(z_i \otimes X_i)\gamma.
\end{aligned}
$$

This final equality follows from the fact that z_i is a single column vector.

Using this expression, we can rewrite $g_N(\gamma)$ and its derivative as

$$g_N(\gamma) = C \cdot \frac{1}{N}\left(\sum_{i=1}^{N} z_i \otimes Y_i - \sum_{i=1}^{N} z_i \otimes X_i \gamma\right)$$
$$\frac{\partial g_N(\gamma)}{\partial \gamma'} = -C \cdot \frac{1}{N}\sum_{i=1}^{N} z_i \otimes X_i.$$

Using these expressions, we can write the first-order condition for minimization of the GMM criterion function as

$$2\left(C \cdot \sum_{i=1}^{N} z_i \otimes Y_i - \left(C \cdot \sum_{i=1}^{N} z_i \otimes X_i\right)\widehat{\gamma}_N\right)' A_N \left(C \cdot \sum_{i=1}^{N} z_i \otimes X_i\right).$$

Rearrainging this yields

$$\widehat{\gamma}_N' \left(C \cdot \sum_{i=1}^{N} z_i \otimes X_i\right)' A_N \left(C \cdot \sum_{i=1}^{N} z_i \otimes X_i\right) = \left(C \cdot \sum_{i=1}^{N} z_i \otimes Y_i\right)' A_N \left(C \cdot \sum_{i=1}^{N} z_i \otimes X_i\right).$$

So the final expression for the GMM estimator is

$$\widehat{\gamma}_N = \left[\left(C \cdot \sum_{i=1}^{N} z_i \otimes X_i\right)' A_N \left(C \cdot \sum_{i=1}^{N} z_i \otimes X_i\right)\right]^{-1} \left(C \cdot \sum_{i=1}^{N} z_i \otimes X_i\right)' A_N \left(C \cdot \sum_{i=1}^{N} z_i \otimes Y_i\right)$$

B.1 The Large-Sample Distribution of $\widehat{\gamma}_N$

To characterize the distribution of the $\widehat{\gamma}_N$, we apply standard asymptotic distributional arguments, letting N go to infinity while holding T fixed. Towards this end, it is straightforward to show that

$$\widehat{\gamma}_N - \gamma_0 = \left[\left(C \cdot \sum_{i=1}^{N} z_i \otimes X_i\right)' A_N \left(C \cdot \sum_{i=1}^{N} z_i \otimes X_i\right)\right]^{-1} \left(C \cdot \sum_{i=1}^{N} z_i \otimes X_i\right)' A_N \left(C \cdot \sum_{i=1}^{N} z_i \otimes u_i(\gamma_0)\right).$$

Using this, we can apply standard cross-sectional asymptotic theory to show that

$$\operatorname*{plim}_{N\to\infty} \widehat{\gamma}_N = \gamma_0,$$

and that

$$\sqrt{N}\left(\widehat{\gamma}_N - \gamma_0\right) \xrightarrow{d} N(0, V),$$

where

$$V = D'^{-1} S D^{-1},$$

$$D = \operatorname*{plim}_{N\to\infty} \frac{1}{N^2} \left(C \cdot \sum_{i=1}^{N} z_i \otimes X_i \right)' A_N \left(C \cdot \sum_{i=1}^{N} z_i \otimes X_i \right),$$

$$S = \operatorname*{plim}_{N\to\infty} \frac{1}{N^3} \left(C \cdot \sum_{i=1}^{N} z_i \otimes X_i \right)' A_N \cdot \left(\sum_{i=1}^{N} C \left(z_i \otimes u_i \left(\gamma_0 \right) \right) \left(z_i \otimes u_i \left(\gamma_0 \right) \right)' C' \right) \cdot A_N \left(C \cdot \sum_{i=1}^{N} z_i \otimes X_i \right).$$

For a given sequence of weighing matrices, D and S can be consistently estimated using their sample analogues.

B.2 The Weighing Matrix

We rely on one-step GMM estimators, calculated using the weighing matrix.

$$A_N = \left(\frac{1}{N} \sum_{i=1}^{N} C \left(z_i z_i' \otimes \Sigma \right) C' \right),$$

where

$$\underset{m\cdot(2\cdot(T-p)-1)\times m\cdot(2\cdot(T-p)-1)}{\Sigma} = \begin{bmatrix} \underset{m\cdot(T-p-1)\times m\cdot(T-p-1)}{A} & \underset{m\cdot(T-p-1)\times m\cdot(T-p)}{0} \\ \underset{m\cdot(T-p)\times m\cdot(T-p-1)}{0} & \underset{m\cdot(T-p)\times m\cdot(T-p)}{I} \end{bmatrix}$$

where

$$A = \begin{bmatrix} 2I_m & -I_m & 0 & 0 & \cdots & 0 \\ -I_m & 2I_m & -I_m & 0 & \cdots & 0 \\ 0 & -I_m & 2I_m & -I_m & \cdots & 0 \\ 0 & 0 & -I_m & 2I_m & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 2I_m \end{bmatrix}.$$

This weighing matrix is the multivariate extension of the initial weighing matrix used by Blundell and Bond (1998).

References

Blundell, Richard and Stephen Bond. (1998) Initial Conditions and Moment Restrictions in Dynamic Panel Data Models. *Journal of Econometrics,* **87**(1), 115–143.