Economics 815 **Winter 2015**

Macroeconomic Theory **Thorsten Koeppl**

Assignment 4

(Due: Tuesday, April 7 – 10 am (in class))

1. Solve Question 3 of Assignment 3 which was postponed.

2. Consider a government purchasing goods from firms. The total level of government consumption is given by

$$G_t = \left(\int_0^1 G_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

and is financed by lump-sum taxes T. The government chooses its demand of individual goods so as to maximize its consumption for any given level of expenditure (and, hence, lump-sum taxes).

For this question, define aggregate demand and the aggregate price index by

$$Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \qquad \text{and} \qquad P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}.$$

(a) Derive the total demand $Y_t(i) = C_t(i) + G_t(i)$ for an individual good i, where private demand is given as in the lecture.

(b) Derive an aggregate market clearing condition for goods and log-linearize the condition around a steady state where the shares of government consumption and private consumption are given by $s_g = G/Y$ and $s_c = C/Y$ respectively.

(c) How do government purchases influence the IS equation? [Hint: Use the market clearing condition in the log-linearized Euler equation and express the output gap again as a deviation of the real interest rate from the natural rate of interest, r_t^n.]

Consider now the following household utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\chi_t C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \right)$$

where χ_t is a preference shock often referred to as a taste or demand shock. Like in the benchmark NK model, the household chooses aggregate consumption C_t optimally over an index of individual goods, can save in nominal one-period bonds which have a price of Q_t, faces lump-sum taxes T_t and supplies labour N_t to firms for a nominal wage equal to W_t. Finally, production in the economy is given by the production function

$$Y(i) = AN_t(i)^{\alpha}.$$

(d) Derive the Euler-equation for the household in terms of aggregate consumption.

(e) Log-linearize the Euler equation and derive an IS equation in terms of a natural rate of interest taking into account that total aggregate demand is given by $Y_t = G_t + C_t$.

(f) Suppose there are no technology shocks. Set $i_t = \rho \equiv -\log \beta$. Show that an appropriately defined fiscal policy can perfectly stabilize the output gap and the inflation rate when χ_t changes over time, but is perfectly and contemporaneously observable by the government.

For this part, please hand in a joint solution with your computational group.

The model is now closed by the standard NK Philips Curve and a reaction function for monetary policy given by

$$\begin{aligned} \pi_t &= \beta E_t[\pi_{t+1}] + \kappa(y_t - y_t^n) \\ i_t &= \bar{\imath} + \phi_\pi \pi_t + \phi_y (y_t - y_t^n), \end{aligned}$$

where the natural level of output associated with flexible prices is given by

$$y_t^n = \psi a_t - \xi,$$

where $\psi = \frac{1+\eta}{\sigma\alpha+\eta+(1-\alpha)}$ and $\xi = \frac{\alpha \log \frac{\epsilon}{\alpha(\epsilon-1)}}{\sigma\alpha+\eta+(1-\alpha)}$.

Finally, consider the following AR(1) processes

$$\begin{aligned} \chi_t &= (1-\rho)\bar{\chi} + \rho_\chi \chi_{t-1} + \epsilon_t \\ g_t &= (1-\rho_g)\bar{g} + \rho_g g_{t-1} + \epsilon_t \end{aligned}$$

where $\rho_i \in (0,1)$ and ϵ_t is an iid shock specific to each process. Choose $\rho_i = 0.95$, use parameter values from Assignment 3 and calibrate any additional parameters.

(g) Use DYNARE to compute IRFs for a taste shock specified with the Taylor-type reaction function for monetary policy. Include $(i_t, \pi_t, r_t - r_t^n, y_t, y_t^n, x_t, c_t)$ and the shock χ_t in your output. [Hint: You can set $\bar{g}$ and $\bar{\chi} = 0$ for the program. Why?]

(h) Use DYNARE to compute IRFs for a government expenditure shock to tastes for the economy specified with the Taylor-type reaction function for monetary policy. Include $(i_t, \pi_t, r_t - r_t^n, y_t, y_t^n, x_t, c_t)$ and your shock g_t in your output.

(i) Now set $\phi_y = 0$ and increase ϕ_π. How do your impulse response functions change? Interpret your results.

3. Consider the following NK model

$$\begin{aligned} \pi_t &= \beta E_t[\pi_{t+1}] + \kappa x_t + u_t \\ x_t &= E_t[x_t] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - \rho) + \epsilon_t \\ i_t &= \rho + \phi_\pi \pi_t. \end{aligned}$$

The two shocks – interpreted as supply and demand shocks – are iid and uncorrelated with variances given by σ_u^2 and σ_e^2 respectively. The long-run steady state values for the output gap and inflation are normalized to 0.

(a) Express the model in matrix form as a system of two linear difference equations for the output gap x_t and inflation π_t.

Bonus: What restrictions on ϕ_π do you need to obtain a stable solution?

(b) Solve for the equilibrium processes of x_t and π_t as a function of the shocks and parameters of the model. [Hint: Iterate forward on the matrix equation.]

Assume now the loss function

$$L = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\alpha x_t^2 + \pi_t^2 \right) \right].$$

Interpret α as a choice parameter for the central bank.

(c) Solve for the value of ϕ_π^* that minimizes the central bank's loss function. [Hint: You need to take as constraints the equilibrium processes for x_t and π_t.]

(d) How does ϕ_π^* depend on the coefficient α, the variances of the shocks and κ? Interpret your results.