Economics 815 **Winter 2014**

Macroeconomic Theory **Thorsten Koeppl**

Assignment 4

(Due: Friday, April 4 – Dropbox until 2 pm)

1. Consider the following household's utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\chi_t C_t^{1-\sigma}}{1-\sigma} + \frac{(1-N_t)^{1-\eta}}{1-\eta} \right)$$

where χ_t is a preference shock often referred to as a taste or demand shock. Like in the benchmark NK model, the household chooses aggregate consumption C_t optimally over an index of individual goods according to

$$C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}.$$

The price for these individual goods are given by $P_t(i)$. The household can save in nominal one-period bonds which have a price of Q_t, faces lump-sum taxes T_t and supplies labour N_t to firms for a nominal wage equal to W_t.

There is a government with aggregate expenditure G_t. This expenditure is financed by lump-sum taxes T_t and is chosen optimally over an index of individual goods.

Production in the economy is given by the production function

$$Y(i) = AN_t(i)^{\alpha}.$$

(a) Set up the household's intertemporal maximization problem.

(b) Derive the Euler-equation for the household in terms of aggregate consumption.

(c) Log-linearize the Euler equation and derive an IS equation in terms of a natural rate of interest taking into account that total aggregate demand is given by $Y_t = G_t + C_t$.

The model is now closed by the standard NK Philips Curve and a reaction function for monetary policy given by

$$\begin{aligned} \pi_t &= \beta E_t[\pi_{t+1}] + \kappa(y_t - y_t^n) \\ i_t &= \bar{\iota} + \phi_\pi \pi_t + \phi_y(y_t - y_t^n), \end{aligned}$$

where the natural level of output associated with flexible prices is given by

$$y_t^n = \psi a_t - \xi,$$

where $\psi = \frac{1+\eta}{\sigma\alpha+\eta+(1-\alpha)}$ and $\xi = \frac{\alpha \log \frac{\epsilon}{\alpha(\epsilon-1)}}{\sigma\alpha+\eta+(1-\alpha)}$.

(d) Suppose there are no technology shocks. Set $i_t = \rho \equiv -\log\beta$. Show that an appropriately defined fiscal policy can perfectly stabilize the output gap and the inflation rate when χ_t changes over time, but is perfectly and contemporaneously observable by the government.

For this part, please hand in a joint solution with your computational group.

Consider the following AR(1) processes

$$\begin{aligned} \chi_t &= (1-\rho)\bar{\chi} + \rho_\chi \chi_{t-1} + \epsilon_t \\ g_t &= (1-\rho_g)\bar{g} + \rho_g g_{t-1} + \epsilon_t \end{aligned}$$

where $\rho_i \in (0,1)$ and ϵ_t is an iid shock specific to each process. Choose $\rho_i = 0.95$, use parameter values from Assignment 3 and calibrate any additional parameters.

(e) Use DYNARE to compute IRFs for a taste shock specified with the Taylor-type reaction function for monetary policy. Include $(i_t, \pi_t, r_t - r_t^n, y_t, y_t^n, x_t, c_t)$ and the shock χ_t in your output. [Hint: You can set $\bar{g}$ and $\bar{\chi} = 0$ for the program. Why?]

(f) Use DYNARE to compute IRFs for a government expenditure shock to tastes for the economy specified with the Taylor-type reaction function for monetary policy. Include $(i_t, \pi_t, r_t - r_t^n, y_t, y_t^n, x_t, c_t)$ and your shock g_t in your output.

(g) Now set $\phi_y = 0$ and increase ϕ_π. How do your impulse response functions change? Interpret your results.

2. Consider an economy where households' preferences are equal to

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\log C_t + \log(1 - N_t) \right) \right]$$

and aggregate production is given by $Y = AN_t^\alpha$.

Also, assume that the economy has monopolistically competitive firms that set their price flexible, so that the aggregate price level P_t is given by

$$P_t = \frac{\epsilon}{\epsilon - 1} \frac{W_t}{MPL_t}$$

where ϵ is the price elasticity of demand, W_t is the nominal wage rate and MPL_t is the marginal product of labour associated with the production function.

(a) Characterize the efficient level of output Y_{SS}^* and the steady state level of natural output Y_{SS}^n associated with monopolistic competition. [Hint: Use the market clearing condition $C = Y$ and the first-order condition with respect to consumption and leisure.]

Assume that there are no productivity shocks, so that there are no fluctuations in the steady state efficient output level, or $Y_t^* = Y_{SS}^*$. Define the new output gap measure $\tilde{x}_t = y_t - y_{SS}^n$, which is the deviation in logs of the actual output from the steady state level of the economy with monopolistic competition. Note that it is different from the output gap $x_t = y_t - y_t^n$.

(b) Express the log-linearized Euler equation of the consumer's problem in terms of the new output gap measure $\tilde{x}_t$ to obtain an IS equation.

(c) Express the New Keynesian Philips Curve in the form

$$\pi_t = \beta E[\pi_{t+1}] + \kappa \tilde{x}_t + \kappa u_t.$$

Interpret the term u_t.

Suppose now the central bank follows a Taylor rule of the form

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{x}_t$$

where $\rho = -\log \beta$, $\phi_\pi = 1.5$ and $\phi_y = 0.5$. Assume further that the economy faces a so-called cost-push shock given by an AR(1) process

$$u_t = \rho_u u_{t-1} + \epsilon_t.$$

For this part, please hand in a joint solution with your computational group.

(d) Set $\rho_u = 0.95$ and use your parameters from the calibration in Assignment 3. Compute IRFs for $i_t, \pi_t, r_t, \tilde{x}_t$ in `DYNARE` for the cost-push shock. Interpret your results.

(e) How does lowering the coefficient ϕ_y influence inflation and the output gap? What conclusions do you draw from this comparative statics exercise for the optimal monetary policy?

Suppose now that

$$u_t = \frac{\lambda}{\kappa}\left[\log\left(\frac{\epsilon_t}{\epsilon_t - 1}\right) - \log\left(\frac{\epsilon_{SS}}{\epsilon_{SS} - 1}\right)\right].$$

(f) Why can you interpret the cost-push shock as a change in the competitiveness in the economy?

(g) What advice would you give the Bank of Canada for its interest rate setting in response to an increase in competitiveness?

3. Consider the following NK model

$$\begin{aligned} \pi_t &= \beta E_t[\pi_{t+1}] + \kappa x_t + u_t \\ x_t &= E_t[x_t] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - \rho) + \epsilon_t \\ i_t &= \rho + \phi_\pi \pi_t. \end{aligned}$$

The two shocks – interpreted as supply and demand shocks – are iid and uncorrelated with variances given by σ_u^2 and σ_e^2 respectively. The long-run steady state values for the output gap and inflation are normalized to 0.

(a) Express the model in matrix form as a system of two linear difference equations for the output gap x_t and inflation π_t.

Bonus: What restrictions on ϕ_π do you need to obtain a stable solution?

(b) Solve for the equilibrium processes of x_t and π_t as a function of the shocks and parameters of the model. [Hint: Iterate forward on the matrix equation.]

Assume now the loss function

$$L = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\alpha x_t^2 + \pi_t^2 \right) \right].$$

Interpret α as a choice parameter for the central bank.

(c) Solve for the value of ϕ_π^* that minimizes the central bank's loss function. [Hint: You need to take as constraints the equilibrium processes for x_t and π_t.]

(d) How does ϕ_π^* depend on the coefficient α, the variances of the shocks and κ? Interpret your results.