Economics 815 **Winter 2015**

Macroeconomic Theory **Thorsten Koeppl**

Answer Key for Assignment 3

Answer to Question 1:

1. The household chooses investment, capital, and consumption to maximize utility. Since they get no utility from leisure, they supply one unit of labour inelastically. Taking into account the uncertainty arising from the AR(1) process describing productivity, the household's decision problem is given by

$$
\begin{aligned}
&\max_{c_t, x_t, k_{t+1}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \right] \\
&\text{subject to} \\
&\quad c_t + \left(1 + \frac{\phi}{2}\frac{x_t}{k_t}\right) x_t = r_t k_t + w_t \\
&\quad k_{t+1} = (1-\delta)k_t + x_t
\end{aligned}
$$

   The Lagrangian is:

$$
L = E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{(1-\gamma)}}{1-\gamma} + \lambda_t \left(r_t k_t + w_t - c_t - x_t - \frac{\phi}{2}\frac{x_t^2}{k_t} \right) + \mu_t (k_t(1-\delta) + x_t - k_{t+1}) \right]
$$

2. The first-order conditions with respect to c_t, k_{t+1} and x_t are given by

$$
\begin{aligned}
&\beta^t c_t^{-\gamma} = \lambda_t \\
&-\mu_t + E_t \left[\mu_{t+1}(1-\delta) + \lambda_{t+1} r_{t+1} + \lambda_{t+1} \frac{\phi}{2} \left(\frac{x_{t+1}}{k_{t+1}} \right)^2 \right] = 0 \\
&-\lambda_t \left(1 + \phi \frac{x_t}{k_t} \right) + \mu_t = 0
\end{aligned}
$$

   Define now the variable $q_t = \frac{\mu_t}{\lambda_t}$. We can rearrange the last FOC so that

$$
q_t = 1 + \phi \frac{x_t}{k_t}.
$$

This equation implies that

$$x_t = \frac{k_t}{\phi}(q_t - 1).$$

Hence, investment is positive whenever q_t is greater than one.

The variable q_t can be related to Tobin's q. The Lagrange multiplier λ_t measure the marginal cost of putting additional investment into place. The multiplier μ_t measures the marginal benefit from having an additional unit of capital. Hence, q_t expresses the value of capital in terms of its (replacement) cost. Without adjustment costs ($\phi = 0$) we have the standard RBC model where q_t is constant and equal to 1.

3. Using the FOC for consumption and dividing by λ_t, the FOC with respect to capital can be rearranged as

$$q_t = \beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \left(F_{kt+1} + \frac{\phi}{2} \left(\frac{x_{t+1}}{k_{t+1}} \right)^2 + q_{t+1}(1-\delta) \right) \right]$$

or

$$q_t = \beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \left(F_{kt+1} + \frac{1}{2\phi} \left(q_{t+1} - 1 \right)^2 + q_{t+1}(1-\delta) \right) \right]$$

where we have used the fact that in equilibrium $r_{t+1} = F_{kt+1}$. Hence, the intertemporal Euler equation has been expressed in terms of capital and Tobin's q.

4. The steady state is characterized by constant consumption, output, investment and capital. Hence, we have that

$$\begin{aligned}
x^* &= \delta k^* \\
q^* &= 1 + \phi\delta \\
F_k^* &= \frac{1}{\beta} q^* - (1-\delta) q^* - \frac{1}{2\phi}(q^* - 1)^2 \\
y^* &= k^{*\alpha} \\
c^* &= y^* - x^* \left(1 + \delta \frac{\phi}{2} \right)
\end{aligned}$$

with factor prices given by

$$\begin{aligned}
r^* &= \alpha F_k^* \\
w^* &= (1-\alpha) F_l^*.
\end{aligned}$$

When $\phi = 0$ this reduces to the standard RBC model, where $q^* = 1$. For $\phi > 0$, we have $q^* > 1$, and using the expression for q^* one can show that

$$\beta [F_k - (1 - \delta)] > 1.$$

Hence, the rate of return is larger in steady state and capital is lower. The intuition is clear. With adjustment costs, it is more expensive to invest for households. They require a larger interest rate in order to be compensated for putting capital into place through costly investment. In equilibrium, due to the extra costs capital will be lower and so are consumption and output.

5. The dynamic system is now described by two non-linear difference equations, one for investment and the intertemporal Euler equation. These equations can be formulated in terms of k_t and q_t. The two figures show the output of `DYNARE` for the case $\phi = 1$ and the benchmark case $\phi = 0$.

   Note that the productivity shock initially increases the rate of return on capital (aka real interest rate) and output 1-1 in percentage deviations initially. Then, capital starts to react.[1] However, when there are adjustment costs of capital/investment, less is invested and more is consumed relative to the RBC benchmark case as the cost of investment increase. Interest rates fall below their long-run steady state level before returning eventually to the their steady state level (which cannot be seen in the graph due to the large persistence parameter on the productivity shock). The reason is that eventually the impact of additionally accumulated capital outweighs the effect of increased productivity. Also, in the benchmark case q is constant at its steady state level which is equal to 1. In the case with adjustment costs, q is first above the long-run steady state level making additional investments attractive, before falling below its steady state level once capital starts to decline. In this case, investment does not fully replace all of the depreciated capital stock anymore.

[1] Recall that `DYNARE` output reports actually the control variable k_{t+1} and not the state variable k_t. This explains the jump in capital in the graphs.

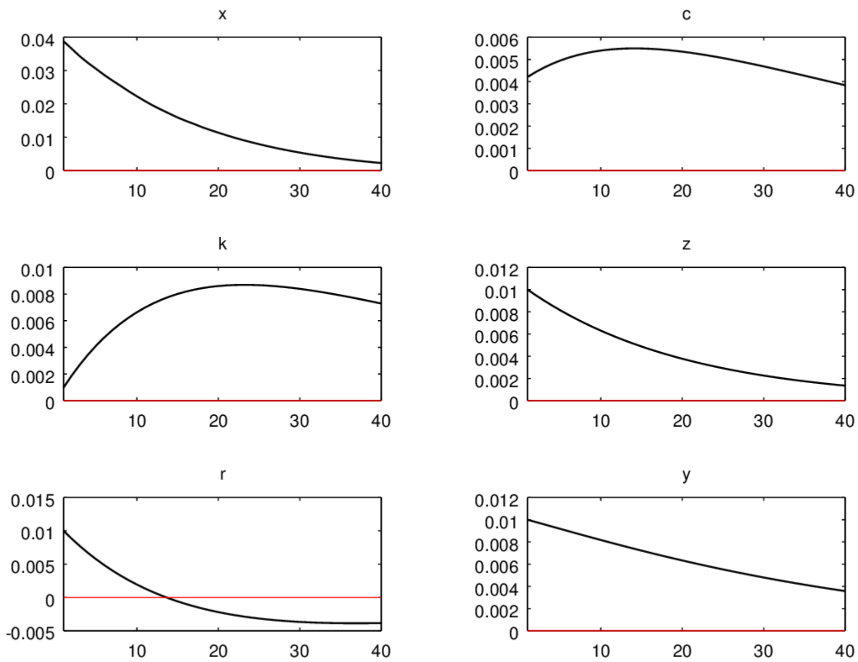

Figure 1: Response to a positive productivity shock (RBC / $\phi = 0$)

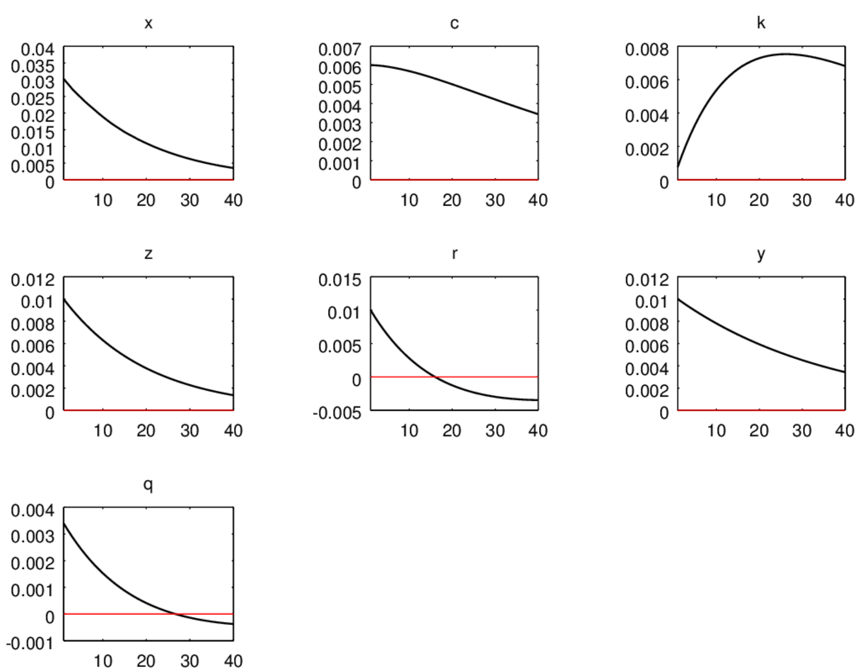

Figure 2: Response to a positive productivity shock ($\phi = 5$)

Answer to Question 2:

1. The choice of data is quite judicious. For inflation, one can use changes in CPI, core inflation or a GDP deflator with the first one being the most natural one. For the output gap, one usually uses unemployment, deviations from long-run unemployment (if we exclude an intercept) or even simply log GDP. However, this measure might not be a good proxy, for example if labour participation rates vary considerably. Finally, if we want to treat inflation expectation as separate from the model, we need to find data for them. One choice are real vs. nominal bonds. The problem is that this spread has a bias and overestimates inflation expectations as real bonds trade at a discount due to a less liquid market. Other possibilities are for example data based on surveys.

   Note that strictly speaking one would need to check stationarity of the time series data first. The price level is clearly non-stationary. Inflation, however, is stationary depending on the time horizon over which the data have been collected. If one chooses a long enough time horizon that includes in particular the 1970s and 1980s, inflation tends to be non-stationary. Hence, one needs to resort to changes in inflation to obtain a stationary time series. Tests for stationarity of the data are quite standard (e.g. Dickey-Fuller test).

2. For the remainder of this question we assume that $\beta \simeq 1$. The regression equation is given by

   $$\pi_t = \beta_1 E_t[\pi_{t+1}] + \beta_2 u_t + \epsilon_t.$$

   Here, we treat inflation expectations as exogenous – and not related to our model of the NK Philips Curve. However, the relationship as exhibited by the NK model presumes that there are rational expectations. This will change in the other parts of this question.

3. We can now rewrite the NK Philips curve to obtain

   $$E_t[\pi_{t+1} - \pi_t] = \kappa(y_t - y_t^n).$$

   With rational expectations we can express this equation as

   $$\pi_{t+1} - \pi_t = \kappa(y_t - y_t^n) + \eta_t$$

where η_t is a one-step ahead forecast error. Suppose now that we estimate then the relationship

$$\pi_{t+1} - \pi_t = \beta_2 u_t + \epsilon_t$$

where ϵ_t now contains the forecast error η_t. It is known from that literature that there could be a problem with OLS estimation, since ϵ_t is likely to be correlated with the variable u_t.

4. Again rewriting the expression, we obtain that

$$E_t[\pi_{t+1} - \pi_t] = \frac{\nu}{1-\nu}(\pi_t - \pi_{t-1}) + \kappa(y_t - y_t^n)$$

so that the regression equation becomes

$$\pi_{t+1} - \pi_t = \beta_1(\pi_t - \pi_{t-1}) + \beta_2(y_t - y_t^n).$$

The discussion is identical to the previous specification. This formulation is called an augmented NK Philips Curve with a backward-looking term for inflation. This term is usually interpreted as some people behaving like as if they had adaptive expectations.

For the results, this is my prior for what you will find – independent of the data you use.

- The "naive" specification should work best. It simply points out that inflation is driven by expected future inflation and that the output gap doesn't matter much.
- The plain vanilla, rational expectation based NK Philips curve has no fit with the data.
- The augmented one (with the backward looking term) gets a better fit, but possibly with the "wrong" sign on the inflation term.

The bottomline is here that in the data there seems to be little confirmation for the validity of the NK Philips curve.

Answer to Question 3:

This question has been postponed to Assignment 4 and the answer will be provided in the answer key to this assignment.