Economics 815 **Winter 2014**

Macroeconomic Theory **Thorsten Koeppl**

Answer Key for Assignment 3

Answer to Question 1:

1. The choice of data is quite judicious. For inflation, one can use changes in CPI, core inflation or a GDP deflator with the first one being the most natural one. For the output gap, one usually uses unemployment, deviations from long-run unemployment (if we exclude an intercept) or even simply log GDP. However, this measure might not be a good proxy, for example if labour participation rates vary considerably. Finally, if we want to treat inflation expectation as separate from the model, we need to find data for them. One choice are real vs. nominal bonds. The problem is that this spread has a bias and overestimates inflation expectations as real bonds trade at a discount due to a less liquid market. Other possibilities are for example Conference Board of Canada data based on surveys.

2. The price level should clearly be non-stationary. Inflation is stationary depending on the time horizon over which the data have been collected. If one chooses a long enough time horizon that includes in particular the 1970s and 1980s, inflation is non-stationary. Hence, one needs to resort to changes in inflation to obtain a stationary time series. Tests for stationarity of the data are quite standard (e.g. Dickey-Fuller test).

3. For the remainder of this question we assume that $\beta \simeq 1$. The regression equation is given by

$$\pi_t = \beta_1 E_t[\pi_{t+1}] + \beta_2 u_t + \epsilon_t.$$

   Here, we treat inflation expectations as exogenous – and not related to our model of the NK Philips Curve. However, the relationship as exhibited by the NK model presumes that there are rational expectations. This will change in the other parts of this question.

4. We can now rewrite the NK Philips curve to obtain

$$E_t[\pi_{t+1} - \pi_t] = \kappa(y_t - y_t^n).$$

With rational expectations we can express this equation as

$$\pi_{t+1} - \pi_t = \kappa(y_t - y_t^n) + \eta_t$$

where η_t is a one-step ahead forecast error. Suppose now that we estimate then the relationship

$$\pi_{t+1} - \pi_t = \beta_2 u_t + \epsilon_t$$

where ϵ_t now contains the forecast error η_t. Of course, there is a problem, since ϵ_t is likely to be correlated with the variable u_t, making OLS estimation inappropriate.

5. Again rewriting the expression, we obtain that

$$E_t[\pi_{t+1} - \pi_t] = \frac{\nu}{1-\nu}(\pi_t - \pi_{t-1}) + \kappa(y_t - y_t^n)$$

so that the regression equation becomes

$$\pi_{t+1} - \pi_t = \beta_1(\pi_t - \pi_{t-1}) + \beta_2(y_t - y_t^n).$$

The discussion is identical to the previous specification. This formulation is called an augmented NK Philips Curve with a backward-looking term for inflation. This term is usually interpreted as some people behaving like as if they had adaptive expectations.

6. For the results, this is my prior for what you will find – independent of the data you use.

   - The 'naive' specification should work best. It simply points out that inflation is driven by expected future inflation and that the output gap doesn't matter much.
   - The plain vanilla, rational expectation based NK Philips curve has no fit with the data.
   - The augmented one (with the backward looking term) gets a better fit, but possibly with the 'wrong' sign on the inflation term.

Answer to Question 2:

1. I chose parameters like the ones in Gali. They are as follows

   - $\theta = 0.66$ – prices on average reset every three quarters[1]
   - $\sigma = \eta = 1$ – standard benchmark
   - $\beta = 0.99$ – about 4% per cent risk-free rate
   - $\epsilon = 6$ – from literature
   - $\alpha = 2/3$ – standard benchmark.

   The parameters for the AR(1) process can be taken from the Solow residual estimation in the first half. Alternatively, one can assume $\rho_a = 0.9$ and use a 1% deviation from steady state as a shock.

   Remark: Calibrating parameters should be done with care in general. However, there is no standard way of carrying this out. Of course, this means that the key parameters like θ or η involve a lot of judgement.

2. For the impulse response function associated with these parameters, see the notes for Lecture XIV.

3. For θ close to 1 (larger price stickiness), the output gap becomes larger as firms cannot adjust their prices. However, higher θ implies lower κ and, hence, lower inflation. As a consequence, the response of policy (nom. interest rates) is muted to the technology shock for the specified Taylor rule.

4. For ϵ close to 1, the output gap is unaffected. However, κ increases. Hence, inflation responds more strongly to the fixed change in the output gap. The incentive to change prices aggressively has increased for firms that are able to do so. This implies that also the nominal interest rate will respond more strongly, even though there are no changes in the output gap.

[1] The expected time to reset is given by $\sum_{t=0}^{\infty} \left(\frac{2}{3}\right)^n \frac{n}{2} = 3$.

Answer to Question 3:

1. Total public demand for good i is given by

$$G_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} G_t.$$

The derivation is of course identical to the one given in the lecture notes for private demand. Total lump-sum taxes T in nominal terms are taking the place of private nominal expenditures Z_t in the derivations. Since taxes are lump-sum, none of the analysis changes.

2. Total demand for good i is given by $C(i) + G(i)$. Define aggregate output by using the following aggregate across demand (and, hence, output) for individual goods

$$Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}.$$

From the individual demand functions it follows that

$$\begin{aligned} Y_t &= \left(\int_0^1 \left[\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} (C_t + G_t)\right]^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= (C_t + G_t)\left(\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} di\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= (C_t + G_t)\left(\frac{1}{P_t}\right)^{-\epsilon}\left(\int_0^1 (P_t(i))^{1-\epsilon} di\right)^{\frac{-\epsilon}{1-\epsilon}} \\ &= C_t + G_t \end{aligned}$$

where we have used the definition of the price index $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$.

Now log-linearizing around the steady state $Y_{SS} = C_{SS} + G_{SS}$ we obtain

$$Y_{SS} \log\left(\frac{Y_t}{Y_{SS}}\right) = C_{SS} \log\left(\frac{C_t}{C_{SS}}\right) + G_{SS} \log\left(\frac{G_t}{G_{SS}}\right).$$

Changing notation and dividing by Y_{SS}, we obtain

$$\hat{y}_t = \frac{C_{SS}}{Y_{SS}}\hat{c}_t + \frac{G_{SS}}{Y_{SS}}\hat{g}_t = s_c\hat{c}_t + s_g\hat{g}_t$$

where s_c and s_g are shares of private and public consumption in steady state.

3. We have that $\hat{c}_t = \frac{1}{s_c}\hat{y}_t - \frac{s_g}{s_c}\hat{g}_t$. Using this in the IS equation, we obtain

$$\frac{1}{s_c}y_t - \frac{s_g}{s_c}g_t = E_t[\frac{1}{s_c}y_{t+1} - \frac{s_g}{s_c}g_{t+1}] - \frac{1}{\sigma}\left(i_t - E_t[\pi_{t+1}] - \rho\right)$$

where we have used the fact twice that $\hat{x}_t = x_t - x_{SS}$.

Multiplying by s_c and rearranging yields

$$y_t = E_t[y_{t+1}] - \frac{s_c}{\sigma}\left(i_t - E_t[\pi_{t+1}] - \rho\right) - s_g E_t[g_{t+1} - g_t].$$

Define now $r_t^n = \rho + \frac{\sigma}{s_c}E_t[y_{t+1}^n - y_t^n]$. Then, we obtain the IS equation in terms of the output gap and the natural rate of interest as

$$x_t = E_t[x_{t+1}] - \frac{s_c}{\sigma}\left(i_t - E_t[\pi_{t+1}] - r_t^n\right) - s_g E_t[g_{t+1} - g_t].$$

Suppose first that gov't spending does not fluctuate across time. This implies that there are no shocks to the IS equation other than through the effect on r_t^n. Of course the fact that private consumption is only a fraction s_c of total demand moderates the impact of changes in interest rates i_t. The reason is that the gov't is not maximizing intertemporally.

Suppose now that gov't spending can fluctuate over time. From a positive perspective, this can be interpreted as an additional source of shocks that influence private demand. Holding everything else fixed, expected changes in gov't spending will lead to changes in the output gap in the same direction. Hence, from a normative perspective, varying gov't expenditure – e.g. for a fixed $i_t = \bar{\imath} = \rho$ – can perfectly stabilize the output gap.