Economics 815 **Winter 2015**

Macroeconomic Theory **Thorsten Koeppl**

Assignment 3

(Due: Monday, March 23 – in class)

1. Consider a basic RBC model without labor-leisure choice. In every period, the utility function for households is given by

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

with the household's endowment of time being normalized to 1. Future utility is discounted according to $\beta \in (0,1)$. The production function is given by

$$y = zk^{\alpha}n^{1-\alpha}$$

where $\log z$ follows an AR(1) process with coefficient ρ and the innovations being normally distributed with zero mean and variance σ^2.

Capital is owned by the household, but is costly to adjust. For investment x_t, the resource costs are given by

$$\left(1 + \frac{\phi}{2}\frac{x_t}{k_t}\right)x_t,$$

where $\phi \geq 0$. Capital evolves according to the law of motion

$$k_{t+1} = k_t(1-\delta) + x_t.$$

(a) Set up the decision problem for the household. [Hint: Use both the budget constraint and the law of motion of capital.]

(b) Define q_t as the ratio of Lagrange-multipliers on the budget constraint and the law of motion on capital. Derive the first-order condition that describes the optimal choice of investment, consumption and capital for the household in terms of q_t. When is investment positive?

(c) Find the intertemporal Euler equation in terms of q_t and q_{t+1}.

(d) Characterize the steady state for this economy. How does it differ from the standard model where $\phi = 0$?

Choose parameters from your earlier calibration and set $\phi = 1$.

(e) Using `DYNARE` compute impulse response functions for i_t, c_t, k_t, y_t and q_t for a positive technology shock.

(f) Compare your impulse response functions to the case where $\phi = 0$ and where $\phi >> 1$. Explain the difference.

2. Consider the New Keynesian Phillips Curve

$$\pi_t = E[\pi_{t+1}] + \kappa(y_t - y_t^n).$$

where π_t is inflation, $E[\pi_{t+1}]$ is expected inflation next period and $y_t - y_t^n$ is a measure of the output gap.

(a) Describe which data (and at what frequency) you are using for inflation, expected inflation and the output gap.

<u>Naive Approach</u>

(b) Express the New Keynesian Phillips curve as a regression equation and estimate it directly. Why is this approach "naive"?

<u>Rational Expectations Approach</u>

(c) Express the New Keynesian Phillips curve as a regression equation taking into account that expectations are rational and estimate it. [Hint: The explained variable is now changes in inflation $\pi_{t+1} - \pi_t$ with the residuals being interpreted as a forecast error.]

Extended Model – Partially Backward-looking

Consider now the extended version of the Phillips curve given by

$$\pi_t = (1-\nu)E_t[\pi_{t+1}] + \nu\pi_{t-1} + \kappa(y_t - y_t^n).$$

(d) Express the extended model again as a regression equation with rational expectations and estimate it.

3. Consider the New Keynesian model described by the equations

$$\begin{aligned}
\pi_t &= \beta E_t[\pi_{t+1}] + \kappa(y_t - y_t^n) \\
y_t - y_t^n &= -\frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - r_t^n) + E_t[y_{t+1} - y_{t+1}^n] \\
i_t &= \rho + \phi_\pi \pi_t + \phi_y(y_t - y_t^n).
\end{aligned}$$

Set the policy parameters so that $\phi_\pi = 1.5$ and $\phi_y = 0.125$. Assume that productivity follows an AR(1) process given by

$$a_t = \rho_a a_{t-1} + \epsilon_t.$$

For the parameters of the productivity process (ρ_a, σ_a) use the values that you have estimated before.

The natural level of output associated with flexible prices is given by

$$y_t^n = \psi a_t - \xi,$$

where $\psi = \frac{1+\nu}{\sigma\alpha+\nu+(1-\alpha)}$ and $\xi = \frac{\alpha \log \frac{\epsilon}{\alpha(\epsilon-1)}}{\sigma\alpha+\nu+(1-\alpha)}$.

This implies that the natural rate of interest is given by

$$r_t^n = \rho + \sigma\psi E_t[a_{t+1} - a_t]$$

(a) Choose parameters $(\alpha, \kappa, \epsilon, \beta, \rho, \sigma, \nu)$ in the Canadian context and justify their values.

Computational Part

(b) Compute IRFs for $i_t, r_t, r_t^n, \pi_t, y_t, y_t^n$ and $x_t = y_t - y_t^n$ in `DYNARE` for a technology shock. Interpret your results.

(c) Now set κ close to 0. How do your IRFs change? Interpret your results.

(d) Now set ϵ close to 1. How do your IRFs change? Interpret your results.