Economics 815 **Winter 2015**

Macroeconomic Theory **Thorsten Koeppl**

Assignment 2

(Due: Friday, February 13 – in class)

1. Consider the following three specifications for technological progress

   - Labour-augmenting: $Y(t) = F(K(t), A(t)L(t))$
   - Capital-augmenting: $Y(t) = F(A(t)K(t), L(t))$
   - Output-augmenting: $Y(t) = A(t)F(K(t), L(t))$

   where labour is constant over time and normalized to 1.

   (a) For each case, find the growth rate for a Balanced Growth Path.

   (b) Suppose that technological progress is labour-augmenting with growth rate g_A and that the labour force also grows at rate g_L. What is the growth rate associated with a Balanced Growth Path?

   (c) Assume that the function F is Cobb-Douglas with shares α for capital and $1-\alpha$ for labour. Show that the three formulations are equivalent subject to a reformulation of the process $A(t)$.

   (d) For the Cobb-Douglas case, are the specifications consistent with the five stylized growth facts? [Hint: Use the labour-augmenting case.]

2. Consider the basic RBC model discussed in lecture. In every period, the utility function for households is given by

$$u(c, 1-n) = \frac{c^{1-\gamma}}{1-\gamma} + \theta \frac{(1-n)^{1-\eta}}{1-\eta}$$

with the household's endowment of time being normalized to 1. Future utility is discounted according to $\beta \in (0,1)$. The production function is given by

$$y = zk^{\alpha}n^{1-\alpha}$$

where $\log z$ follows an AR(1) process with coefficient ρ and the innovations being normally distributed with zero mean and variance σ^2. Finally, assume that capital depreciates with rate δ.

(a) Derive the steady state conditions in terms of (k, c, y).

(b) Calibrate your economy to data from the Canadian economy as discussed in class. In particular, pick θ such as to match a target for the fraction of time spent working. Carefully describe your strategy for calibration and the data that you are using. [Hint: Do not calibrate, but set $\gamma = \eta = 1$.]

<u>Computational Part</u>

(c) Compute steady state values for (k, c, y) and θ. How well do the ratios c/y, k/y and x/y match the corresponding ratios in the data?

(d) Use `DYNARE` to compute impulse response functions for labour, consumption, capital and output. What correlations do you obtain between labour and output as well as between consumption and output?

(e) Adjust your parameters to $\gamma = 5$, $\eta = 0$ and recalibrate θ to the same target. Report how your answers to part (d) change and interpret these changes.

3. Consider the following two period economy. There is a household with preferences over consumption and leisure as specified in the previous question, but without discounting ($\beta = 1$). The household has an endowment of time equal to 1 in each of the two periods. Households can save or borrow from each other at the gross interest rate $1 + r$.

   There is also a firm that carries out production according to

$$y_t = An_t^{\alpha}.$$

There is also a government that needs a total of g resources to build useless pyramids at the end of the two periods. The only way for the government to obtain resources is to tax people's labour income. It can levy taxes (τ_1, τ_2) to obtain revenue, but cannot borrow or lend. The government needs to ensure that it raises sufficient taxes to cover g over time. The resources it obtains in each period are not longer available for consumption, so that $c_t = y_t - g_t$.

(a) Define a competitive equilibrium for this economy for any given policy (g, τ_1, τ_2).

(b) Formulate the government's budget constraint *in each period*, distinguishing between g_1 and g_2.

(c) Find the first-order conditions for the household's problem and the firm's problem as a function of taxes.

<u>Computational Part:</u>

Set $\gamma = \eta = 2$, $\alpha = 0.5$ and $A = 1$. Fix $g = 0.2$ as the government's expenditure.

(d) Compute the labour supply (n_1, n_2) and revenue $(g_1, g_2$ as a function of government policy τ_1. Plot your results. [Hint: Express the FOCs and the relationship $g_1 + g_2 = g$ as a function of $(n_1, n_2, \tau_1, \tau_2)$ only. Then solve numerically for (n_1, n_2, τ_2) for different values of τ_1. Note that there will be limits on what values τ_1 can take.]

(e) Find the tax rate τ_1 that maximizes the household's utility. What does this tax rate imply for the time path of government revenue g_1 and g_2? Interpret your answer.

(f) Now suppose the government would like to maximize its overall revenue g by setting taxes $\tau = \tau_1 = \tau_2$. Plot the associated revenue g as a function of τ and find the revenue maximizing value of the tax τ and the associated revenue g for the government.