Assignment 2

(Due: Friday, February 14 – Dropbox until 3 pm)

1. Consider the basic RBC model discussed in lecture. In every period, the utility function for households is given by

$$u(c, 1 - n) = \frac{c^{1-\gamma}}{1 - \gamma} + \theta \frac{(1 - n)^{1-\eta}}{1 - \eta}$$

with the household's endowment of time being normalized to 1. Future utility is discounted according to $\beta \in (0, 1)$. The production function is given by

$$y = zk^{\alpha}n^{1-\alpha}$$

where $\log z$ follows an AR(1) process with coefficient ρ and the innovations being normally distributed with zero mean and variance σ^2. Finally, assume that capital depreciates with rate δ.

   (a) Derive the steady state conditions in terms of (k, c, y).

   (b) Calibrate your economy to data from the Canadian economy as discussed in class. In particular, pick θ such as to match a target for the fraction of time spent working. Carefully describe your strategy for calibration and the data that you are using. [Hint: Do not calibrate, but set $\gamma = \eta = 1$.]

   Computational Part

   (c) Compute steady state values for (k, c, y) and θ. How well do the ratios c/y, k/y and x/y match the corresponding ratios in the data?

   (d) Use DYNARE to compute impulse response functions for labour, consumption, capital and output. What correlations do you obtain between labour and output as well as between consumption and output?

(e) Adjust your parameters to $\gamma = 5$, $\eta = 0$ and recalibrate θ to the same target. Report how your answers to part (d) change and interpret these changes.

2. Consider the following two period economy. There is a household with preferences over consumption and leisure as specified in the previous question, but without discounting ($\beta = 1$). The household has an endowment of time equal to 1 in each of the two periods. Households can save or borrow from each other at the gross interest rate $1 + r$.

There is also a firm that carries out production according to

$$y_t = An_t^\alpha.$$

There is also a government that needs a total of g resources to build useless pyramids at the end of the two periods. The only way for the government to obtain resources is to tax people's labour income. It can levy taxes (τ_1, τ_2) to obtain revenue, but cannot borrow or lend. The government needs to ensure that it raises sufficient taxes to cover g over time. The resources it obtains in each period are not longer available for consumption, so that $c_t = y_t - g_t$.

(a) Define a competitive equilibrium for this economy for any given policy (g, τ_1, τ_2).

(b) Formulate the government's budget constraint *in each period*, distinguishing between g_1 and g_2.

(c) Find the first-order conditions for the household's problem and the firm's problem as a function of taxes.

Computational Part:

Set $\gamma = \eta = 2$, $\alpha = 0.5$ and $A = 1$. Fix $g = 0.05$ as the government's expenditure.

(d) Compute the labour supply n_1 and n_2 as a function of government policy τ_1. Plot your result. [Hint: Use the FOCs, the market clearing conditions and the fact that $g_2 = g - g_1$, where g_1 is a function of τ_1. Note that there will be limits on what values τ_1 can take.]

(e) Plot the revenue g_1 and g_2 as a function of τ_1.

(f) Find the tax rate τ_1 that maximizes the household's utility. What does this tax rate imply for the time path of government revenue g_1 and g_2? Interpret your answer.

(g) Now suppose the government would like to maximize its overall revenue g by setting taxes $\tau = \tau_1 = \tau_2$. Plot the associated revenue g as a function of τ and find the optimal value of the tax τ and the associated revenue g for the government.