ECON 815

Long-run Growth

Winter 2015

Balanced Growth Path

Definiton: An economy is on a balanced growth path if all variables grow at the same rate.

We have some well established facts that we would like to capture.

- Y/L and K/L grow over time.
- K/Y is constant.
- Wages increase over time
- Interest rates remain constant.
- Income shares remain constant over time.

Some Accounting

Dynamic evolution given by feasibility:

$$Y = C + X = C + K' + (1 - \delta)K$$

Hence: $g_Y = g_C = g_K$.

Output given by some production function

$$Y = F(K, L)$$

We have

$$g_Y = \frac{K}{Y}\frac{\partial F}{\partial K}g_K + \frac{L}{Y}\frac{\partial F}{\partial L}g_L$$

Hence:

$$(1 - \eta_K)g = \eta_L g_L$$

With constant-return-to-scale, we obtain that $\eta_K + \eta_L = 1$ and, thus, $g = g_L$.

There must be some source of exogenous growth for the economy to evolve along a BGP with positive growth.

This source affects effective labor – or equivalently – labour productivity (*labor-augmenting* technological progress).

One can easily verify that the neoclassical production function is consistent with the stylized growth facts.

BGP and Detrending

Suppose Y_t, C_t and K_t all grow at the same rate γ.

Detrending, we obtain

$$
\begin{aligned}
y_t &= \frac{Y_t}{(1+\gamma)^t} \\
c_t &= \frac{C_t}{(1+\gamma)^t} \\
k_t &= \frac{K_t}{(1+\gamma)^t}
\end{aligned}
$$

Then, we are dealing with a stationary economy that is described by

$$
\begin{aligned}
y_t &= k_t^{\alpha} \\
y_t &= c_t + (1+\gamma)k_{t+1} - (1-\delta)k_t \\
u(c_t) &= \frac{c_t^{1-\sigma}}{1-\sigma}(1+\gamma)^{t(1-\sigma)}
\end{aligned}
$$

where the population size has been normalized.

Social planning problem

$$\max_{c_t,k_t} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}(1+\gamma)^{t(1-\sigma)}$$

subject to

$$k_t^\alpha = c_t + (1+\gamma)k_{t+1} - (1-\delta)k_t$$

Solution:

$$\frac{1}{\beta}\left(\frac{c_t}{c_{t+1}}\right)^{-\sigma}(1+\gamma)^\sigma = (1-\delta) + \alpha k_{t+1}^{\alpha-1}$$

This Euler equation governs the transition to a BGP that is given by

$$\alpha \bar{k}^{\alpha-1} = \frac{1}{\beta}(1+\gamma)^\sigma - (1-\delta)$$

What governs long-run interest rates?

Suppose we have

$$\begin{aligned} Y_t &= A_t K_t^{\alpha} N_t^{1-\alpha} \\ &= A_0 K_t^{\alpha} \left[(1+\mu)^{\frac{t}{1-\alpha}} (1+n)^t N_0 \right]^{1-\alpha} \end{aligned}$$

where $A_0 = N_0 = 1$.

The (exogenous) growth rate is approximately given by

$$\gamma \simeq \frac{\mu}{1-\alpha} + n$$

In the BGP, interest rates must be constant and are given by

$$r = \alpha \bar{k}^{\alpha-1}$$

$$
\begin{aligned}
r &= \frac{(1+\gamma)^{\sigma}}{\beta} - (1-\delta) \\
&\simeq \log\beta + \sigma\gamma + \delta \\
&\simeq \theta + \delta + \sigma\left(\frac{\mu}{1-\alpha} + n\right)
\end{aligned}
$$

For nominal interest rates that we mostly observe, we can use the Fisher equation which is given by

$$1 + r = \frac{1+i}{1+\pi^{e}}$$

or

$$i \simeq r + \pi^{e}$$

where π^{e} is expected inflation.