ECON 815

A Basic New Keynesian Model II

Winter 2015

Unemployment vs. Inflation

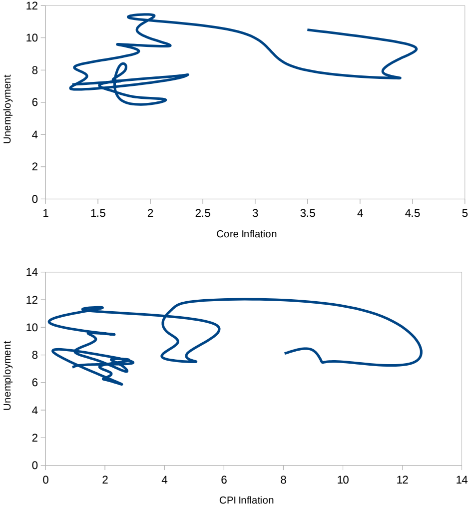
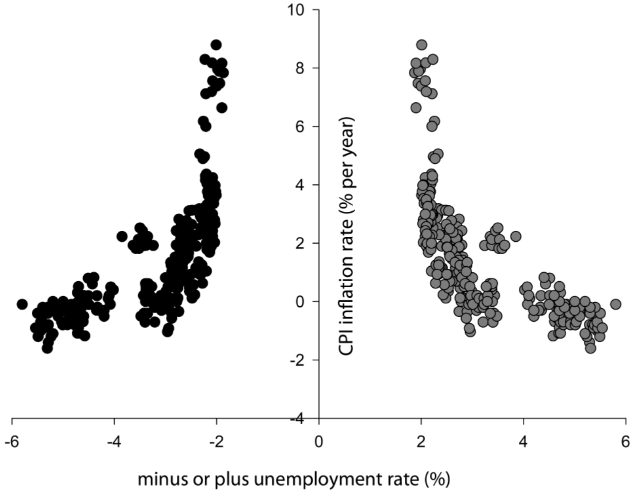


Figure : Unemployment vs. Inflation – Canada 1977 - 2013

Japan


10
8
6
4
2
0
-2
-4
CPI inflation rate (% per year)
-6
-4
-2
0
2
4
6
minus or plus unemployment rate (%)

Sticky Prices

Firms can change their price only with probability $(1-\theta)$.

- Suppose they set their price in period $t=0$.
- In period $t=1$, with probability θ they cannot change it.
- In period $t=2$, conditional on not having changed in period 1, they cannot change it with probability θ.
- And so on ...

Firms solve:

$$\max_{P_t(i)} \sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} \left(P_t(i) Y_{t+k}(i) - W_{t+k} \left(\frac{Y_{t+k}(i)}{A_{t+k}} \right)^{\frac{1}{\alpha}} \right) \right]$$

subject to

$$Y_{t+k}(i) = \left(\frac{P_t(i)}{P_{t+k}} \right)^{-\epsilon} C_{t+k}$$

where $Q_{t,t+k}$ captures "stochastic equilibrium discounting".

FOC:

$$\sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} Y_{t+k}(i) \left(P_t(i) - \frac{\epsilon}{\epsilon - 1} \varphi_{t+k}(i) \right) \right] = 0$$

All firms that can change prices today, will chose the same price, P_t^*.

When the price cannot be adjusted in period $t+k$, the firm sets labour demand to satisfy its demand for goods

$$N_{t+k}(i) = \left(\frac{Y_{t+k}(i)}{A} \right)^{\frac{1}{\alpha}} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\frac{\epsilon}{\alpha}} \left(\frac{C_{t+k}}{A} \right)^{\frac{1}{\alpha}}$$

Hence, firm cannot maintain its *desired mark-up* μ when its price is sticky.

Distortion

For fixed P_t^*, we have $\partial N_{t+k}(i)/\partial P_{t+k} > 0$.

Then, we have that nominal marginal costs $\varphi_{t+k}(i)$ are large.

We have that the mark-up is depressed, i.e. $\mu_t < \mu$, or equivalently that real marginal costs are high.

Unexpected price increases will lead to higher labour demand. Hence, mark-ups can be interpreted as a labour wedge that depresses the MPL.

Conclusion:

Firms would like to increase prices with the price level, but cannot do so. However, they set prices that take into account expected changes in nominal marginal costs.

Steady State

Suppose we have zero inflation in steady state, $P_t^* = P_{t-1}$ so that there is no need to change prices.

The following equations describe the steady state

$$\begin{aligned} Y(i) &= Y \\ Y &= AN^\alpha \\ -\frac{U_n}{U_c} &= \alpha A \frac{N^{\alpha-1}}{P} \\ 1 &= \beta(1+\bar{\iota}) \\ P &= \frac{\epsilon}{\epsilon-1}\varphi(i) \end{aligned}$$

Hence, firms charge identical mark-up and real marginal costs are constant at $1/\mu$.

We now look at equilibrium in a log-linearized version around this steady state.

Aggregate Price Level

The price index in period t is given by

$$P_t^{1-\epsilon} = \int_{i|fixed} P_{t-1}(i)^{1-\epsilon} di + (1-\theta) P_t^{*1-\epsilon}$$

The distribution of fixed prices corresponds to the distribution of last periods prices with weight θ, or

$$\theta P_{t-1}^{1-\epsilon} = \int_{i|fixed} P_{t-1}(i)^{1-\epsilon} di$$

Hence, inflation is given by

$$\Pi_t = \left[\theta + (1-\theta) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$$

Inflation changes less than 1-1 with price changes of individual firms.

Inflation Dynamics

We look at approximations around a zero inflation steady state.

The log-linearized firm's FOC and the inflation equation give

$$\begin{aligned} \pi_t &= \beta E_t[\pi_{t+1}] + \lambda \left(\log \frac{\bar{\varphi}_t}{P_t} - \log \frac{\epsilon - 1}{\epsilon} \right) \\ &= \lambda \sum_{t=0}^{\infty} \beta^k E_t \left[\log \frac{\bar{\varphi}_{t+k}}{P_{t+k}} - \log \frac{\epsilon - 1}{\epsilon} \right] \end{aligned}$$

where

- $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{\alpha}{\alpha+\epsilon(1-\alpha)}$
- $\frac{\bar{\varphi}_t}{P_t}$ are average real marginal costs for firms

Inflation is given by *expected* deviations from steady-state mark-up.

Inflation is high (low) whenever firms expect real marginal costs above (below) their steady state values.

The New Keynesian Philips Curve

We can express deviations in real marginal costs in terms of deviations in output.

$$\log \frac{\bar{\varphi}_t}{P_t} - \log \frac{\epsilon - 1}{\epsilon} = \left(\sigma + \frac{\eta + (1-\alpha)}{\alpha}\right)(y_t - y_t^n)$$

The last expression is the *output gap* which measures the deviation of the actual output level from the (optimal) output level associated with flexible prices, y_t^n.

This links inflation dynamics to dynamics in output, or

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa(y_t - y_t^n)$$

where $\kappa = \lambda \left(\sigma + \frac{\eta+(1-\alpha)}{\alpha}\right)$.

Summary – The NK Trinity

1) Phillips Curve

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa(y_t - y_t^n)$$

2) Intertermporal Euler equation

$$y_t - y_t^n = -\frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - r_t^n) + E_t[y_{t+1} - y_{t+1}^n]$$

where $r_t^n = \rho + \sigma E_t[y_{t+1}^n - y_t^n]$ is the natural rate of interest which changes due to real (or supply) shocks.

3) Specification of a monetary policy rule.

The Phillips Curve specifies inflation in terms of the output gap which is given by the Euler equation through the natural rate and the actual real rate. The latter one is pinned down by monetary policy.