ECON 815

Two-Period Economy

Winter 2014

Basic Model Set-up

- one good for period 1 (good 1) and period 2 (good 2)
- endowment y_1 and y_2
- people (measure 1) have utility over consumption of these goods

$$u(c_1, c_2) = u(c_1) + \beta u(c_2)$$

- competitive markets for trading these goods

Equilibrium:

A set of prices (p_1, p_2) and an allocation (c_1, c_2) such that

(i) people maximize utility taking prices as given

(ii) markets clear, i.e. $c_1 = y_1$ and $c_2 = y_2$.

Solution:

$$\frac{u'(y_1)}{\beta u'(y_2)} = \frac{p_1}{p_2}$$

Adding Savings

Suppose people can save endowments, earn interest rate r and consume the receipts in the second period.

Three markets – for goods in each period and a market for saving and borrowing.

People solve the following problem:

$$\max u(c_1) + \beta u(c_2)$$
$$\text{subject to}$$
$$c_1 + s = y_1$$
$$c_2 = y_2 + (1+r)s$$

Equilibrium:

An interest rate r and an allocation (c_1, c_2, s) s.th.

(i) people maximize utility taking the interest rate as given

(ii) markets clear, i.e. $c_1 = y_1$, $c_2 = y_2$ and $s = 0$.

Equivalence

Solution:

$$\frac{u'(y_1)}{\beta u'(y_2)} = (1+r)$$

Note that

$$\frac{p_1}{p_2} = (1+r)$$

Hence, both ways are equivalent with interest rates being a ratio of *intertemporal prices*.

Suppose further that $y_1 = y_2$. We simply get $(1+r) = 1/\beta$ (or in terms of a discount rate $r = \theta$).

Adding Production and Investment

Firms:

- have technology and are owned by people
- borrow goods x from people in period 1
- costlessly convert these goods x into capital k
- pay back goods plus interest $(1+r)x$ in period 2

Production:

$$f(k) + (1-\delta)k$$

Assumptions:

- $f(0) = 0$, $f'(0) = \infty$ and $f'(\infty) = 0$
- $f' > 0$ and $f'' < 0$

Firms maximize profits:

$$\max_k \Pi = \max_k f(k) + k(1-\delta) - (1+r)k$$

People obtain these profits (here in goods in period 2).

<u>Equilibrium:</u>

An interest r and an allocation (x, k, c_1, c_2) such that

(i) people maximize utility taking the interest rate as given

(ii) firms maximize profits taking the interest rate as given

(iii) markets clear, i.e. $c_1 = y_1$, $c_2 = y_2$ and $x = k$.

For profit maximization to have a solution that corresponds to an equilibrium, we need to have that

$$f'(k) + (1 - \delta) = (1 + r)$$

or

$$f'(k) = r + \delta$$

Solution:

$$\text{MRT} = f'(k) + (1 - \delta) = (1 + r) = \frac{p_1}{p_2} = \frac{u'(c_1)}{\beta u'(c_2)} = \text{IMRS}$$

We now have $c_1 = y_1 - k$ and $c_2 = y_2 + f(k) + (1 - \delta)$.

Issue: One (nonlinear) equation in one unknown variable k. Need computation.

Example

- $u(c) = \ln c$
- $f(k) = k^\alpha$ and $\delta = 0$
- $y_1 = y$ and $y_2 = 0$

Let's look first at a *social planner solution*.

This solution simply picks an allocation that maximizes utility for people, while respecting all (technological) constraints.

$$\max_k \ln(y - k) + \beta \ln(k^\alpha)$$

<u>Solution:</u>

$$\begin{aligned} c_1 &= \frac{1}{1+\alpha\beta} y \\ k &= \frac{\alpha\beta}{1+\alpha\beta} y \\ c_2 &= \left(\frac{\alpha\beta}{1+\alpha\beta} y \right)^\alpha \end{aligned}$$

Key insight: Solution depends on parameters (α, β, y) and the model structure.

Let's use our first-order condition:

$$\text{MRT} = \alpha k^{\alpha-1} = \frac{c_2}{\beta c_1} = \text{IMRS}$$

Using the market clearing conditions, we obtain exactly the same result.

This is just a consequence of the two fundamental theorems of welfare economics.

Where do we stand?

So far, we have a DGE model.

To go to a DSGE model that is more interesting for macro, we need to add uncertainty and shocks ...

... and some other "stuff".