ECON 815

Financial Shocks I

Winter 2014

Investment in the RBC Model

Output can be transformed 1-1 into capital.

- price of capital must be identical to the price of consumption
- supply of capital is perfectly price-elastic

Demand for capital depends on the marginal product of capital.

- productivity shock shifts demand
- output follows AR(1) process after shock
- no persistence of shock

Modelling Investment

Idea: Investment requires borrowing.

Moral hazard leads to financial frictions with borrowing.

1) Commitment Problem

- need to secure borrowing
- how? collateral
- even though returns are high, cannot obtain funds

2) Default Problem

- investment is risky
- private information on returns
- bankruptcy is costly
- incentives require (partly) inside funding

Key: moral hazard limits outside funding.

Model of Financial Contracting

Risk-neutral borrower:

- produces investment goods
- turns i units of goods into ωi units of capital
- ω is random, mean 1 and distribution Φ
- has own inputs given by n (“net worth”)
- sells output at price q

Risk-neutral lender:

- supplies inputs $i - n$
- obtains return $(1 + r^i)(i - n)$
- cannot observe ω, but verify at a cost μ per unit of investment

Interpretation – Risky debt contract with default if ω is too low and bankruptcy costs μ.

Debt Contract

Set $\bar{\omega} = (1 + r^i)\left(\frac{i-n}{i}\right)$.

Contracts set amount of investment i and the default threshold $\bar{\omega}$ (interest rate).

Borrower has convex pay-off.

- if $\omega \leq \bar{\omega}$: 0
- if $\omega > \bar{\omega}$: $\omega - \bar{\omega}$

Lender has concave pay-off.

- if $\omega \leq \bar{\omega}$: $\omega - \mu$
- if $\omega \geq \bar{\omega}$: $\bar{\omega}$

Total value of *expected* output of capital per unit of investment

$$q\left(\alpha_b(\bar{\omega}) + \alpha_\ell(\bar{\omega})\right) = q(1 - \Phi(\bar{\omega})\mu).$$

Lender obtains expected return equal to his outside option of not investing.

$$q i \alpha_b(\bar{\omega}) = (i - n)$$

Borrower obtains all excess surplus.

$$q i \alpha_\ell(\bar{\omega}) = q i(1 - \Phi(\bar{\omega})\mu) - (i - n).$$

Investment is thus given by

$$i = n \left(\frac{1}{1 - q\alpha_b(\bar{\omega})} \right)$$

There is leverage

$$q\alpha_\ell(\bar{\omega}) \frac{i}{n} = q\alpha_\ell(\bar{\omega}) \left(\frac{1}{1 - q\alpha_b(\bar{\omega})} \right)$$

which increases with q.

Investment Supply

Expected output of investment good per borrower is given by

$$i(q,n)\,(1-\Phi(\bar{\omega})\mu)$$

Assume $\bar{\omega}$ is fixed so that the shares α_b and α_ℓ are fixed.

Output of investment goods is an increasing function in q and net worth n shifts the function $i(q,n)$ for given q.

Aggregating over all borrowers (η of them) yields the investment supply function

$$I^s = \eta i(q,n)\,(1-\Phi(\bar{\omega})\mu) = Z\left(\frac{1}{1-q\alpha_b(\bar{\omega})}\right)(1-\Phi(\bar{\omega})\mu)$$

where $Z = \eta n$ is total net worth in the economy.

Adjustment Costs of Capital

Household Problem:

$$\max_{c,k,i,n} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{c(t)^{1-\sigma}}{1-\sigma} + \chi \frac{(1-n_t)^{1-\eta}}{1-\eta} \right) \right]$$

subject to

$$c_t + i_t + \phi(i_t) \leq w_t n_t + r_t k_t$$
$$k_{t+1} = (1-\delta)k_t + i_t$$

where $\phi' > 0$, $\phi'' \geq 0$ and $\phi'(0) = 0$.

FOC:

$$1 + \phi'_t = \frac{\mu_t}{\lambda_t} \equiv q_t$$
$$q_t c_t^{-\sigma} = E_t \left[\beta c_{t+1}^{-\sigma} (r_{t+1} + (1-\delta) q_{t+1}) \right]$$

Tobin's q

q_t is called *Tobin's q*.

- market value of capital relative to the costs of capital in terms of consumption
- μ_t – marginal value in terms of utility of one more unit of capital tomorrow
- λ_t – marginal costs of one more unit of capital in terms of foregone consumption
- if $\phi' > 0$, there are adjustment costs of capital
- hence: μ_t/λ_t needs to be larger than 1 for $i_t > 0$