ECON 815

A Classical "Monetary" Economy

Winter 2014

Households

$$\max_{C_t, N_t, B_t, M_t/P_t} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\left(\frac{M_t}{P_t}\right)^{1-\nu}}{1-\nu} + \frac{(1-N_t)^{1-\eta}}{1-\eta} \right) \right]$$

subject to

$$P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t - T_t \quad \text{for all } t$$

$$\lim_{T \to \infty} E_t[B_T] = 0 \quad \text{for all } t$$

- T are transfers (government, profits)
- $Q_t = \frac{1}{1+i_t}$ is the *nominal* bond price
- W_t is the nominal wage
- M_t is nominal money holdings

Using $A_t = B_t + M_t$ for financial assets, we have

$$P_t C_t + Q_t A_t + (1 - Q_t) P_t \frac{M_t}{P_t} \leq A_{t-1} + W_t N_t - T_t$$

Firms

There is no capital.

Taking prices and wages as given, firms solve

$$\max_{N_t} P_t Y_t - W_t N_t$$
$$\text{subject to}$$
$$Y_t = A N_t^{\alpha}$$
$$\log A_t = \rho \log A_{t-1} + \epsilon_t$$

where $\alpha \in (0,1)$ and $\epsilon_t \sim \mathbb{N}(0, \sigma_\epsilon^2)$.

FOC:

$$\frac{W(s^t)}{P(s^t)} = \alpha A(s^t) N(s^t)^{\alpha - 1}$$

FOCs & Money Demand

$$
\begin{aligned}
\frac{M_t(s^t)}{P_t(s^t)} &= C(s^t)^{\frac{\sigma}{\nu}} \left(\frac{i_t}{1+i_t}\right)^{-\frac{1}{\nu}} \\
\frac{C(s^t)^\sigma}{(1-N_t)^\eta} &= \frac{W_t}{P_t} \\
1 &= \beta E_t \left[\frac{C(s^t)^\sigma}{C(s^{t+1})^\sigma} \frac{P(s^t)}{P(s^{t+1})}(1+i_t)\right]
\end{aligned}
$$

The first equation is money demand. Real balances

- increase with output (income)
- decrease with the nominal interest rate.

The Fisher Equation

For savings, the (expected) *real interest rate* matters

$$(1 + i_t(s^t))\frac{P(s^t)}{P(s^{t+1})} = (1 + i_t)\frac{1}{1 + \pi(s^{t+1})} = (1 + r(s^{t+1}))$$

or in logs

$$r(s^{t+1}) = i_t - \pi(s^{t+1}).$$

Log-linearizing, the intertemporal Euler equation becomes

$$0 = \log\beta - \sigma E_t[c_{t+1}] + \sigma c_t - E_t(\pi_{t+1}) + i_t$$

or

$$c_t = E_t[c_{t+1}] - \frac{1}{\sigma}\left(i_t - \bar{r} - E_t[\pi_{t+1}]\right)$$

The term in brackets expresses deviations of (expected) real interest rate r_t from the steady state interest rate $\bar{r}$.

Why is there money?

In the background, there is some service money provides that bonds cannot.

- Cash-in-Advance constraint
- Search models
- Turnpike and OG models

Money is costly. Why? Forgive interest rate for bonds, in order to hold money (rate-of-return dominance).

Hence, people would like to economize on holding real balances as much as possible.

The Friedman Rule

What would the optimal monetary policy be? Minimize the opportunity cost for holding money.

Hence, set zero nominal interest rates ($i_t = 0$ for all t).

This implies that prices have to decline at the discount rate

$$\frac{P_{t+1}}{P_t} = \beta$$

or – equivalently – that there is deflation according to the rate of time preference

$$\pi = -\rho (= -r)$$

where we have used logs.

Monetary Policy

There can be two *instruments*:

- money supply M_t
- nominal interest rate i_t

Money supply rules would pin down all nominal variables through the money demand relationship.

- theoretically – we obtain a determinate path for the price level
- empirically – difficulties in controlling inflation

Interest rate rules allows us to forget about money altogether, but we need to be careful about determinancy.

Why? Once i_t (and, consequently, equilibrium variables) has been chosen, money supply simply adjusts to satisfy the money demand equation.

Interest Rate Rules and Indeterminancy

Consider now a policy that sets $i_t = \bar{\iota} = \bar{r}$ for all t.

From the Fisher equation, it must be the case that $E_t[\pi_{t+1}] = \bar{r} - r_t$.

Hence, expected inflation is pinned down by shocks that influence r_t, but actual inflation is not.

Take for example any stochastic process such that $E_t[\epsilon_{t+1}] = 0$ that influences the price level $t+1$ for all t. This is consistent with the Fisher equation.

Conclusion: Price level and actual inflation is indeterminate.

Feedback Rules and the Taylor Principle

Consider now the rule $i_t = \bar{\iota} + \phi_\pi \pi_t$.

Interpret $\bar{\iota}$ as a neutral, nominal interest rate consistent with an inflation target. For example, $\bar{\iota} = \bar{r} + 2\%$.

The Fisher equation now satisfies

$$\pi_t = \frac{1}{\phi_\pi} \left(E_t[\pi_{t+1}] + r_t - \bar{\iota} \right).$$

If $\phi_\pi > 1$ (**Taylor Principle**), we can iterate forward to obtain a unique stationary solution. This solution only depends on $E_t[r_{t+n}]$.

Otherwise, we again have indeterminancy.

Heroic Assumption: We assume $\phi_\pi > 1$ and pick the stationary solution and live with it!