Liquidity and Crises in Financial Markets

Risk Management II

U of Basel, HS 2012

Brunnermeier and Pedersen (2009)

Does the liquidity of asset markets depend on the ease of financing a position?

- Yes. Funding liquidity puts constraints on taking up positions.
- Channel is through both, margin requirements and losses.
- Asset markets are linked through this channel and co-move.
- More risky and illiquid assets are relatively more affected.

Hence, prices reflect all three aspects, fundamental value, initial selling/buying pressure and funding constraints.

Prices can move discontinuously in response to initial shocks (amplification, crises).

Basic Idea

Shock – initial selling pressure increases.

This pushes prices downward.

Usually, speculators are profiting from lower prices and stabilize the market.

Idea 1:

If the initial price drops and they make losses, their funding might be compromised and there is a loss spiral.

Idea 2:

If the price drop is misinterpreted (!) as lower fundamental values and increased price risk, margin requirements will increase. Then, margins increase, speculators reduce their positions and there is a margin spiral.

Model

There are three periods with trading only at $t = 1, 2$.

There is a single asset with an uncertain pay-off v_3.

There is some random demand for securities in $t = 1$ and $t = 2$.

Speculators alleviate trading pressure, but are constrained by having to fund their positions.

Uncertainty

Fundamental value v_t follows an ARCH process:

$$v_{t+1} = v_t + \sigma_t \epsilon_{t+1}$$

ϵ_{t+1} is iid across time and is distributed acc. to $\mathcal{N}(0, 1)$.

Volatility follows an autoregressive process

$$\sigma_{t+1} = \sigma + \theta |v_{t+1} - v_t|$$

Current shocks increase future volatility ($\theta > 0$) and initial values for $t = 0$ are given

Demand Side

Investors:

- have initial wealth W_0
- no endowment of assets
- preferences: $u(W_t) = -e^{-\gamma W_t}$

Investor 1:

- has endowment shock $z(1)$ of the asset in $t = 3$
- can trade both at $t = 1$ and $t = 2$

Investor 2:

- has endowment shock $z(2)$ of the asset in $t = 3$
- can trade only at $t = 2$

Assumption: Balanced market at $t = 2$, $z(1) + z(2) = 0$.

Demand at $t = 2$

Investors wealth tomorrow is given by

$$W_3 = W_2 + (v_3 - p_2)(y_2(i) + z(i))$$

Demand:

$$y_2(i) = \frac{v_2 - p_2}{\gamma \sigma_3^2} - z(i)$$

All investors are present.

No order imbalance, as they perfectly insure each other:

$$v_2 = p_2 \text{ and } y_2(i) = -z(i)$$

Hence: No uncertainty left for W_3, as net position in the asset is 0 for everyone.

Demand at $t = 1$

Now only investor 1 is around and there is a temporary demand imbalance.

He knows that $W_2 = W_3$. His final wealth is thus given by

$$W_2 = W_1 + (p_2 - p_1)(y_1(1) + z(1))$$

Same demand function:

$$y_1(1) = \frac{v_1 - p_1}{\gamma \sigma_2^2} - z(1)$$

Speculators make up the supply and in equilibrium some price p_1 is realized such that markets clear.

Supply Side

Speculators are risk neutral and choose position x_1 to maximize their expected wealth.

Their wealth evolves according to

$$W_t = W_{t-1} + (p_t - p_{t-1})x_{t-1} + \eta_t$$

where η_t is a wealth shock.

They face a capital constraint on their positions

$$|x_1|m_1(\text{sign}(x)) \leq W_1$$

This can be interpreted as a funding constraint.

Supply at $t = 1$

Note first that $E_1[v_2] = E_1[p_2] = v_1$.

Hence:

- if $v_1 > p_1$, go long: $|x_1| = W_1/m_1(+)$
- if $v_1 < p_1$, go short: $|x_1| = W_1/m_1(-)$

Total expected gross profits:

$$E_1[W_2] = W_1 + (E_1[v_2] - p_1)x_1 = W_1 \left(1 + \frac{v_1 - p_1}{m(\text{sign}(x))}\right)$$

Key insight:

Position restricted by margins and, hence, there is a relationship between equilibrium prices and margins.

Risk Management

We assume a basic VaR model with parameter π.

Hence:

$$\pi = \Pr(-(p_2 - p_1) > m_1(+))$$
$$\pi = \Pr((p_2 - p_1) > m_1(-))$$

We look at two different scenarios:

1. Margins are set with full information.
2. Margins are set assuming that $v_t = p_t$ for all t.

We analyze a situation with long positions for speculators at $t = 1$ and denote the margin simply by m_1.

This corresponds to a situation where investor 1 has $z(1) > 0$ and would like to sell the asset to hedge his risk.

Case 1 – Stabilizing Margins

Suppose first that margins are set with full information at $t = 1$.

Then, we have that

$$\pi = 1 - \Phi\left(\frac{m_1 + (v_1 - p_1)}{\sigma_2}\right)$$

Hence:

$$m_1 = \Phi^{-1}(1 - \pi)\left(\sigma + \theta|v_1 - v_0|\right) - (v_1 - p_1)$$

where Φ is the cdf of the standard normal distribution.

We view the deviation of the price from fundamentals $(v_1 - p_1)$ as a measure of illiquidity.

Margins are a function of

- risk management π
- underlying (average) volatility σ and current shocks ϵ_1 (or v_1)
- illiquidity in the market $(v_1 - p_1)$

Tighter risk management and bigger current shocks to fundamentals increase margins.

But risk management recognizes here when prices are low due to investor demand. This lowers margins.

Intuition:

On average speculators will make profits tomorrow and, hence, risk on the position declines.

Case 2 – Destabilizing Margins

Now margins are set according to $v_t = p_t$.

Then, we have that

$$\pi = 1 - \Phi\left(\frac{m_1}{\sigma_2}\right)$$

This is equivalent of basing margins on backward looking price volatility estimates:

$$(p_2 - p_1) \sim \mathcal{N}(0, (\sigma + \theta|p_1 - p_0|)^2)$$

Rewriting, we obtain that

$$\begin{aligned} m_1 &= \Phi^{-1}(1-\pi)\,(\sigma + \theta|(v_1 - v_0) - [(v_1 - p_1) - (v_0 - p_0)]|) \\ &= \Phi^{-1}(1-\pi)\,(\sigma + \theta|(v_1 - v_0) - (v_1 - p_1)|) \end{aligned}$$

since I assume that $v_0 = p_0$.

Consider no fundamental shock, but an increase in selling pressure.

- $v_1 = v_0$, $\sigma_2 = \sigma$ and $(v_1 - p_1) > 0$
- p_1 falls
- interpreted as increase in fundamental volatility due to a fundamental shock
- increase in long position by speculators
- but increase in margins
- less funding leads to less demand, p_1 falls further

Margins will be higher whenever selling pressure and bad news happen at the same time.

Margins will be lower when liquidity shocks and fundamental shocks offset each other.

Response of Equilibrium Prices

What is the total effect of shocks on *equilibrium* prices under the two scenarios for the setting of margins?

In equilibrium:

$$W_1 = W_0 + (p_1 - p_0)x_0 + \eta_1 = m_1 x_1 = m_1(-y_1(1)) = m_1 \left(z(1) - \frac{v_1 - p_1}{\gamma \sigma_2^2} \right)$$

We consider a shock to speculators' capital (η_1) and selling pressure $z(1) > 0$ (other shocks HW!).

Totally differentating, we get

$$\frac{\partial p_1}{\partial \eta_1} = \frac{1}{\frac{1}{\gamma \sigma_2^2} m_1 + \frac{\partial m_1}{\partial p_1} x_1 - x_0}$$

The total effect depends on $\partial m_1 / \partial p_1$ and the comovement of speculator and investor positions.

Margin Spirals

Assume $x_0 = 0$.

For constant margins we get the standard response

$$dp_1 = \frac{1}{\frac{1}{\gamma\sigma_2^2} m_1} d\eta_1$$

For stabilizing margins (informed), we get

$$\frac{\partial m_1}{\partial p_1} = 1$$

so that margins are reduced when p_1 falls.

We get that

$$dp_1 = \frac{1}{\frac{\Phi^{-1}(1-\pi)}{\gamma\sigma} + z(1)} d\eta_1$$

Price decreases depend only on exogenous parameters and price impact is moderated ($z(1) > 0$).

For destabilizing margins (uninformed), we look at the case where $v_1 = v_0$.

We have that

$$\frac{\partial m_1}{\partial p} = \Phi^{-1}(1-\pi)\theta(-1) < 0$$

so that margins increase, when prices fall.

Price changes are not related to fundamental causes and we obtain

$$dp_1 = \frac{1}{\frac{\Phi^{-1}(1-\pi)}{\gamma\sigma} - \Phi^{-1}(1-\pi)\theta z(1)} d\eta_1$$

Hence, price decreases are amplified.

The effect is greater for more stringent risk management (lower π) and more reinforcing volatility (higher θ).

Loss Spiral

Suppose now that $x_0 > 0$.

Then, $z(1) > 0$ leads to a price decrease which is reinforced through a loss on the initial position x_0.

If $x_0 < 0$, the opposite is true, since now speculators gain from their initial short position.

Example – loss effect with informed risk management

$$dp_1 = \frac{1}{\frac{\Phi^{-1}(1-\pi)}{\gamma\sigma_2} + z(1) - x_0} d\eta_1$$

Also: If $x_0 > z(1)$, the loss effect outweighs the stabilizing effect through margins.

What is essential here?

1. Market making activity needs to be constrained by capital.

2. Margins need to move sufficiently procyclical.
   - backward looking risk management
   - sufficient noise in price changes
   - initial losses on market making positions

3. There must be (expected) persistence in volatility changes.