Liquidity and Crises in Financial Markets

Risk Management I

U of Basel, HS 2012

Risk Management Basics

In financial trades, one is exposed to *counterparty or default risk*, i.e. the risk that a counterparty does not honor its obligations.

This mostly concerns *price or replacement cost risk*, i.e. the potential loss stemming from closing out or replacing a transaction.

In general, both the likelihood and the potential, but uncertain size of a loss matter.

Common instruments to manage this risk are:

- position limits
- counterparty restrictions
- collateral (margin, haircuts)

Value-at-Risk (VaR)

VaR is the most common risk metric besides volatility or β.

It is used also for the measurement of tail risk for a given distribution of returns.

Denote V_t the value of a portfolio at time t.

Formal definition of VaR as a risk measure:

$$\mathrm{VaR}(\pi) = \inf\{L \in \mathbb{R} | \Pr\left(-(V_{t+\tau} - V_t)\right) > L) < \pi\}$$

Formal definition of VaR as a risk metric:

$$\Pr\left(-(V_{t+\tau} - V_t) \geq \mathrm{VaR}\right) \leq \pi$$

Risk management must determine VaR (and the underlying distribution) and α.

This is then translated into collateral requirements (margins, haircuts) for trading.

Problems:

- distributional assumptions (fat tails)
- parameters usually ad-hoc
- what's the horizon?
- what about jumps (black swans)?
- are losses relevant or something else (bankruptcy costs, collateral costs, reputation, utility)?
- how should one take into account counterparty quality?

Garleanu and Pedersen (2007)

Market liquidity should matter for risk management.

If markets are less liquid, it takes longer to unwind a position.

Hence, in risk management based on VaR, one cannot take the time-horizon as fixed.

New concept:
LVaR – Liquidity adjusted Value at Risk.

This implies that trading positions decrease with illiquidity, since the risk of price movements increases with time.

Main Result

Suppose now that everyone uses LVaR.

Upon an initial shock, everyone will try to unwind positions.

Taking this into account, it takes much longer to do so.

LVaR needs to be tightened and a multiplier effect arises.

This can be viewed as a coordination failure that arises from a very sensible, individual risk management technique.

A similar spiral can arise, whenever risk management induces selling pressures that tighten risk management constraints further.

Model

Model based on a version of DGP.

Measure 1 of risk-neutral investors discount future at r.

S assets with stochastic dividends.

Two types of investors:

- h-type has no holding cost
- l-type has a holding cost δ per unit of asset
- switch type acc. to probabilities (λ_u, λ_d)

Trade opportunities arise acc. to a Poisson process with arrival rate λ.

Key difference: asset holdings are restricted by a risk management constraint.

Uncertainty

Dividends $X(t)$ follow a stochastic process.

Increments are stationary and independently distributed.

Expected return is normalized to 0

$$E_t[X(t+T) - X(t)] = 0$$

Volatility is proportional to time

$$Var_t[X(t+T) - X(t)] = \sigma_X^2 T$$

Risk Management Constraint

Investors can hold a position up to some number $\bar{\theta}$.

This number is determined by satisfying

$$Var_t[\theta(P(X(t+T) - P(X(t))] \leq \sigma^2$$

Note: when X follows a Brownian Motion, one can map this 1-1 into a VaR constraint.

We assume that:

- for h-types, $\sigma = \bar{\sigma}$ sufficiently large
- for ℓ-types: $\sigma = 0$

We look at two forms of this constraint:

1. standard "VaR": based on constant (exogenous) time
2. "LVaR": based on the expected time to sell

Equilibrium

With risk neutrality:

- h-types will hold $\bar{\theta}$ units of securities
- ℓ-types will not hold any securities

Hence, in equilibrium:

$$\begin{aligned}\bar{\sigma}^2 &= Var_t[\bar{\theta}(P(X(t+T) - P(X(t))] \\ S &= \bar{\theta}(1 - \mu_{hn} - \mu_{ln}) \\ 0 &= f(\mu_{hn}|\bar{\theta})\end{aligned}$$

where f is a quadratic equation determined from the law of motion of μ conditional on $\bar{\theta}$.

For an equilibrium, we need to determine three equilibrium objects jointly:

- $P(X(t))$
- $\bar{\theta}$
- μ_{hn}

Some Preliminary Results

An increase in the meeting frequency lowers the number of potential trades at any point in time.

$$\frac{\partial \mu_{lo}}{\partial \lambda} > 0 \text{ and } \frac{\partial \mu_{hn}}{\partial \lambda} > 0$$

Relaxed position limit reduces the selling pressure.

$$\frac{\partial \mu_{lo}}{\partial \bar{\theta}} < 0 \text{ and } \frac{\partial \mu_{hn}}{\partial \lambda} > 0$$

Value functions and Prices

Define a *per unit of asset* value functiom $v(t)$ that is *net* of stochastic dividend flow (and wealth) by

$$\bar{\theta} v_i(t) = \tilde{V}(t) - \mathbf{1}_{\{i \in \{ho, lo\}\}} \frac{X(t)}{r} \bar{\theta}$$

Then, v takes out the stochastic variation in the value function that is due to dividend fluctuations.

Hence: we are back to the framework of DGP with non-stochastic return for the asset equal to 0.

Prices are again determined by Nash-Bargaining with the whole quantity $\bar{\theta}$ being sold:

$$\begin{aligned} P(X(t)) &= \frac{X(t)}{r} + p \\ p &= (v_{lo} - v_{ln})(1-q) + (v_{ho} - v_{hn})q \end{aligned}$$

$$\begin{aligned} rv_{lo} &= -\delta + \lambda_u(v_{ho} - v_{lo}) + 2\lambda\mu_{hn}(p - v_{lo} + v_{ln}) \\ rv_{ln} &= \lambda_u(v_{hn} - v_{ln}) \\ rv_{ho} &= \lambda_d(v_{lo} - v_{ho}) \\ rv_{hn} &= \lambda_d(v_{ln} - v_{hn}) + 2\lambda\mu_{ho}(v_{ho} - v_{hn} - p) \end{aligned}$$

This yields prices given by

$$P(X(t)) = \frac{X(t)}{r} + p = \frac{X(t)}{r} - \frac{\delta}{r} f(r, q, \lambda, \lambda_d, \lambda_u, \mu_{lo}, \mu_{hn})$$

Illiquidity discount depends only indirectly on $\bar{\theta}$:

- it increases with μ_{lo}
- it decreases with μ_{hn}

Risk Management in Equilibrium

With a standard VaR, we obtain in equilibrium

$$\bar{\theta} = \frac{\bar{\sigma}}{\sigma_X} r \frac{1}{\sqrt{\tau}}$$

The larger the horizon, the lower the position limit.

With a LVaR, we obtain instead

$$\bar{\theta} = \frac{\bar{\sigma}}{\sigma_X} r \sqrt{2\lambda\mu_{hn}}$$

The larger the expected time to sell, the lower the position limit.

Comparative Statics

Benchmark: equilibrium with $\tau = 1/(2\lambda\mu_{hn})$.

Hence: the two risk management regimes are exactly the same.

We look at two shocks:

- risk shock – $\bar{\sigma}/\sigma_X$
- "liquidity" – λ

Result: Effects are amplified by LVaR.

Risk Shock

Suppose the variance of dividends σ_X increases (or $\bar{\sigma}$ falls).

The position limit $\bar{\theta}$ needs to decrease.

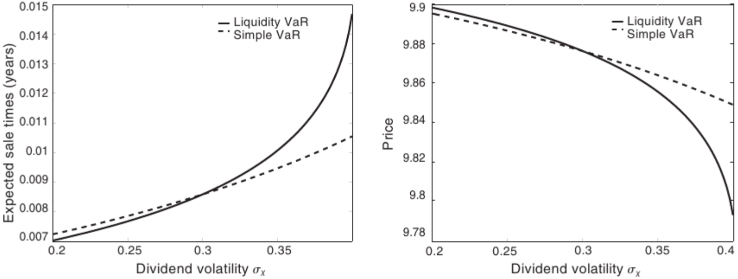
This implies that selling pressure increases and search time also increases:

- μ_{hn} decreases
- μ_{lo} increases

Prices need to fall, as the illiquidity discount increases.

With LVaR, there is an additional effect, since an increase in search time $(1/(2\lambda\mu_{hn})$ tightens $\bar{\theta}$ further:

$$\bar{\theta}_{LVaR} = a\sqrt{2\lambda\mu_{hn}} < a\frac{1}{\sqrt{\tau}} = \bar{\theta}_{VaR}$$



0.015
0.014
0.013
0.012
0.011
0.01
0.009
0.008
0.007
Expected sale times (years)
0.2
0.25
0.3
0.35
Dividend volatility σ_x
Liquidity VaR
Simple VaR
9.9
9.88
9.86
9.84
9.82
9.8
9.78
Price
0.2
0.25
0.3
0.35
0.4
Dividend volatility σ_x
Liquidity VaR
Simple VaR

“Liquidity” Shocks

Consider a technology shock so that λ falls.

We have no effect in a simple VaR.

Why?

- time horizon for the VaR is fixed
- liquidity shocks do not influence the variance of dividends

However: the expected selling time increases.

Hence, with LVaR, we see a negative effect on position limits and prices.

Policy Issues

Risk Management can be procyclical and amplify price movements.

This is partly a coordination failure among market participants.

In principle, better coordination can reduce price swings.

Why not delegate risk management to a third party such as a clearinghouse?

Potential trade-off: better coordination, but more risk due to information disadvantage.