

# Liquidity and Crises in Financial Markets

## OTC Markets II

U of Basel, HS 2012

So far we have:

- frictional trading in OTC markets leads to illiquidity
- price includes an illiquidity discount
- turnover is slow

There is a role for intermediation.

## Extending the model

unit mass of dealers or market makers (MM)

investors meet MM with Poisson intensity  $\rho$

MM are outside option for buying and selling assets

Interdealer market:

- MM can continuously buy and sell assets
- MM cannot hold inventories
- all MM are identical

## Trading patterns

Dealers trade among each other at price  $M$ .

MM buy from low owners with frequency  $\rho\mu_{lo}$  at the ask price  $A$ .

MM sell to non-owners with frequency  $\rho\mu_{hn}$  at the bid price  $B$ .

In steady state  $s < \mu_h$ , so that there is excess demand for assets.

Total dealer transactions are then given by

$$\rho \min\{\mu_{lo}, \mu_{hn}\} = \rho\mu_{lo}.$$

Flows need to be adjusted:

$$\dot{\mu}_{lo}(t) = -2\lambda\mu_{hn}(t)\mu_{lo}(t) - \rho\mu_{lo} - \lambda_u\mu_{lo}(t) + \lambda_d\mu_{ho}(t)$$

Value functions needs to be adjusted:

$$\begin{aligned} V_{lo}(t) = & \frac{1}{1+r\Delta} [(1-\delta)\Delta + \lambda_u\Delta V_{ho}(t+\Delta) \\ & + 2\lambda\mu_{hn}(t+\Delta)\Delta(V_{ln}(t+\Delta) + P) + \rho\Delta(V_{ln}(t+\Delta) + B) \\ & + (1-\lambda\Delta - \rho\Delta - 2\lambda\mu_{hn}(t+\Delta))V_{lo}(t+\Delta) + o(\Delta)] \end{aligned}$$

Hence:

$$rV_{lo} = (1-\delta) + \lambda_u(V_{ho} - V_{lo}) + 2\lambda\mu_{hn}(P + V_{ln} - V_{lo}) + \rho(V_{ln} + B - V_{lo}) + \dot{V}_{lo}$$

Homework: derive other flows and value functions.

## Prices

Nash Bargaining with bargaining power summarized by  $z \in [0, 1]$  for MM.

Ask price

$$A = (V_{ho} - V_{hn})z + M(1 - z)$$

Bid price

$$B = (V_{lo} - V_{ln})z + M(1 - z)$$

## Interdealer Market Equilibrium

We have that  $M = A$ .

- excess demand by dealers:  $\mu_{hn} > \mu_{lo}$
- thus dealers that meet sellers make positive profits:  $M > B$

Prices for sellers depend on relative bargaining power

$$P \geq B \text{ if and only if } z \geq (1 - q)$$

Homework: Prove the last relationship.

## Comparative Statics

Bid-ask spread  $A - B$  is

- increasing in  $z$
- decreasing in  $\lambda$
- (tends to) decreasing in  $\rho$ .

Idea: when search frictions decrease, MM loses bargaining power due to *sequential competition*.

Do we achieve Walrasian pricing?

- for  $\rho \rightarrow \infty \dots$
- ... unless  $z = 1$ .

Why? Transactions tend to be always with monopolistic MM. Hence, bilateral trading on the market is less of an outside option.



## Liquidity Crisis

Weill (2007) – Leaning against the Wind

Idea: market makers take away pressure by building up inventories.

Selling pressure (crisis): initial state is  $\mu_{lo}(0) + \mu_{ln}(0) = 1$ .

The law of motion is now given by

$$\dot{\mu}_h(t) = \lambda_u - (\lambda_u + \lambda_d)\mu_h(t),$$

with  $\mu_h(0) = 0$ .

For  $t \rightarrow \infty$ , we have  $\mu_h(t) \rightarrow \mu_h(SS) = \frac{\lambda_u}{\lambda_u + \lambda_d}$ .

## Dealers and Inventories

Investors can now trade only with MM.

MM:

- zero marginal utility for holding the asset ( $\delta = 1$ )
- hold inventory  $I(t) \geq 0$
- meet investors at rate  $\rho$

Hence:  $\mu_{lo}(t) + \mu_{ho}(t) + I(t) = s$

Profits by buying low, selling high over time.

Competition leads to zero expected profits over time.

Inventory change:

$$\dot{I}(t) = u_l(t) - u_h(t)$$

Gross flows:

$$\begin{aligned} -\rho\mu_{ln}(t) &\leq u_l(t) \leq \rho\mu_{lo}(t) \\ -\rho\mu_{ho}(t) &\leq u_h(t) \leq \rho\mu_{hn}(t) \end{aligned}$$

## Why are inventories wasteful?

Welfare function:

$$\int_0^{\infty} e^{-rt} (\mu_{ho}(t) + (1 - \delta)\mu_{lo}(t)) dt$$

Socially optimal to allocate as many assets to  $h$  types.

But: Opportunity cost with inventories, since MM value assets less.

## Optimal Inventories

Suppose  $I(t) = 0$ .

We have  $\mu_{lo}(t) + \mu_h(t) - \mu_{hn}(t) = s$ .

Then, there is a crossing time  $t_s$  such that  $\mu_{hn}(t) < \mu_{lo}(t)$  for  $t < t_s$  (more sellers) and reverse if  $t > t_s$  (more buyers).

Idea:

- build up  $I$  before  $t_s$
- sell of  $I$  after  $t_s$
- trade-off between better allocation and deadweight cost

## Intuition

MM purchases some additional securities at  $t_s - \Delta$  and sells them off again at  $t_s + \Delta$ .

Feasible? Yes.

- At  $t_s - \Delta$ , we have  $\rho\mu_{lo}(t_s - \Delta) > \rho\mu_{hn}(t_s - \Delta)$ .
- At  $t_s + \Delta$ , we have  $\rho\mu_{lo}(t_s + \Delta) < \rho\mu_{hn}(t_s + \Delta)$ .

Opportunity cost of inventory:

$$(1 - \delta)2\Delta$$

Expected utility cost of search for *lo* type:

$$\delta \frac{1}{\lambda_u + \rho}$$

## Buffer allocations

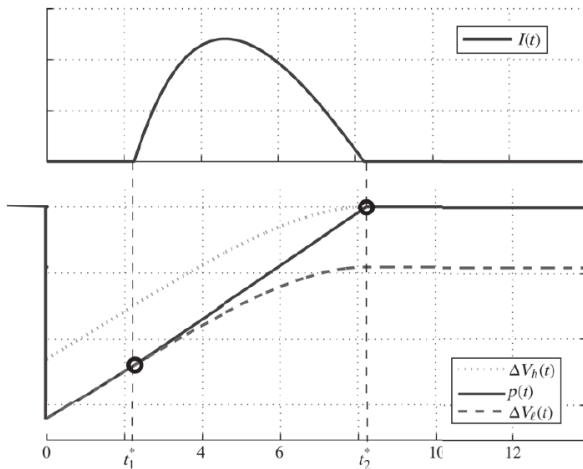


FIGURE 4

The equilibrium price path

## When are inventories optimal?

At  $t = 0$ , it is not necessarily optimal to intervene.

- extreme excess supply
- takes a long time to get rid of an additional unit of inventory
- deadweight cost large
- “match making”:  $u_l(t) = u_h(t) = \rho\mu_{hn}(t)$

At  $t$  large, it is never optimal to intervene.

- excess demand
- no need to hold excess inventories
- “matchmaking”:  $u_l(t) = u_h(t) = \rho\mu_{lo}(t)$

Conclusion: Dealer activity is optimal only around critical time  $t_s$ .



## Competitive MM

MM now buys  $u_\ell(t)$  and sells  $u_h(t)$  at a given price  $p(t)$ .

$$\max_{a, I, c, u_\ell, u_h} \int_0^\infty e^{-rt} c(t) dt$$

subject to

$$\dot{a}(t) = ra(t) + p(t)\dot{I}(t) - c(t)$$

$$\dot{I}(t) = u_\ell(t) - u_h(t)$$

$$a(t), I(t) \geq 0$$

$$a(0) \geq 0 \text{ and } I(0) = 0 \text{ given}$$

$a(t)$  can be interpreted as the capital of a MM.

Can competitive MM achieve the efficient allocation?

## Equilibrium

Prices depend on the marginal investor.

*Price changes* over time need to take into account the incentives to trade or to delay trade.

Price dynamics differ across three regions: excess supply, inventories, excess demand

### Result:

If there is enough capital ( $a(0)$  is sufficiently large), competition among MM achieves the optimal allocation and MM make zero expected profits.

▶ GRAPH

Region 1:  $t \in [0, t_1]$

- $lo$  is the marginal investor:  $p(t) = V_{lo}(t) - V_{ln}(t)$
- MM has no incentive to build inventories.

$$\dot{p}(t) < rp(t) - (1 - \delta) < rp(t)$$

Region 2:  $t \in [t_1, t_2]$

- MM is the marginal investor:  $p(t) > V_{lo}(t) - V_{ln}(t)$
- ... and needs to make zero profits from inventories.

$$\dot{p}(t) = rp(t)$$

Region 3:  $t \in [t_2, \infty)$

- $hn$  is the marginal investor:  $p(t) = V_{hn}(t) - V_{ho}(t)$
- MM has no incentives to build inventories.

$$\dot{p}(t) = 0 < rp(t)$$

Finally, note that MM makes zero profits over all intervals.