Liquidity and Crises in Financial Markets

OTC Markets I

U of Basel, HS 2012

Overview

We look at search models and asset pricing (i.e. trading).

Duffie, Garleanu and Pedersen:

- people search for counterparties
- bargain sequentially

Weill:

- exogenous shock to liquidity
- dealers alleviate price pressure

Chiu and Koeppl:

- opaqueness can lead to adverse selection
- asset purchases in response to market breakdown

Duffie, et al.

Investors – measure 1

discount future at rate r

own one asset or none $\{o, n\}$

fraction s of investors hold an asset

two valuations of the asset $\{h, l\}$

normalized to 1 and a holding cost δ

Markov process instantaneous switching probabilities $\{\lambda_u, \lambda_d\}$

Consumption-based asset pricing (Lucas 1978)

long-run distribution of types

$$\mu_h = \frac{\lambda_u}{\lambda_u + \lambda_d}$$
$$\mu_\ell = \frac{\lambda_d}{\lambda_u + \lambda_d}$$

Walrasian (Price-taking) Equilibrium

If $s \leq \frac{\lambda_u}{\lambda_u+\lambda_d}$, we have that

$$P = \int_{t=0}^{\infty} e^{-r(s-t)} ds = \frac{1}{r}$$

Pretty boring!

Frictional Trading

random search

meetings acc. to a Poisson process with arrival rate λ

bilateral trade with Nash bargaining about the price P

non-owners with high valuations are buyers

owners with low valuation are sellers

Approach:

1. law of motion on investor characteristics
2. characterize the steady state distribution of investors
3. set up value functions as a function of bargaining outcome
4. solve Nash bargaining outcome

Law of motion:

$$\begin{aligned}
\dot{\mu}_{\ell o}(t) &= \underbrace{-2\lambda\mu_{hn}(t)\mu_{\ell o}(t)}_{\text{outflow trade}} \quad \underbrace{-\lambda_u\mu_{\ell o}(t)}_{\text{outflow valuation switch}} \quad \underbrace{+\lambda_d\mu_{ho}(t)}_{\text{inflow valuation switch}} \\
\dot{\mu}_{hn}(t) &= -2\lambda\mu_{hn}(t)\mu_{\ell o}(t) + \lambda_u\mu_{\ell n}(t) - \lambda_d\mu_{hn}(t) \\
\dot{\mu}_{ho}(t) &= 2\lambda\mu_{hn}(t)\mu_{\ell o}(t) + \lambda_u\mu_{\ell o}(t) - \lambda_d\mu_{ho}(t) \\
\dot{\mu}_{\ell n}(t) &= 2\lambda\mu_{hn}(t)\mu_{\ell o}(t) - \lambda_u\mu_{\ell n}(t) + \lambda_d\mu_{hn}(t)
\end{aligned}$$

$$\begin{aligned}
\mu_{lo}(t) + \mu_{ho}(t) &= s \\
\mu_{lo}(t) + \mu_{ln}(t) + \mu_{ho}(t) + \mu_{hn}(t) &= 1
\end{aligned}$$

In the steady state, some assets are not allocated optimally.

We still have

$$\mu_{hn} - \mu_{lo} = \frac{\lambda_u}{\lambda_u + \lambda_d} - s > 0,$$

with $\mu_{lo} > 0$.

Without search frictions (Lucas), we would have $\mu_{hn} = \frac{\lambda_u}{\lambda_u+\lambda_d} - s$.

Value functions

$$\begin{aligned}
rV_{lo} &= 1 - \delta + \lambda_u(V_{ho} - V_{lo}) + 2\lambda\mu_{hn}(P + V_{ln} - V_{lo}) + \dot{V}_{lo} \\
rV_{ln} &= \lambda_u(V_{hn} - V_{ln}) + \dot{V}_{ln} \\
rV_{ho} &= 1 + \lambda_d(V_{lo} - V_{ho}) + \dot{V}_{ho} \\
rV_{hn} &= \lambda_d(V_{ln} - V_{hn}) + 2\lambda\mu_l o(V_{ho} - P - V_{hn}) + \dot{V}_{hn}
\end{aligned}$$

How do we derive these?

Method 1:

1. approximate Poisson processes for a small time interval $[t, t+\Delta]$

$$
\begin{aligned}
V_{lo}(t) = \frac{1}{1+r\Delta} & [(1-\delta)\Delta + \lambda_u \Delta V_{ho}(t+\Delta) \\
& + 2\lambda\mu_{hn}(t+\Delta)\Delta(V_{ln}(t+\Delta)+P) \\
& \quad +(1-\lambda\Delta - 2\lambda\mu_{hn}(t+\Delta)V_{lo}(t+\Delta) + o(\Delta)]
\end{aligned}
$$

2. rewriting we obtain

$$
\begin{aligned}
r\Delta V_{lo}(t) = (1-\delta)\Delta + \lambda_u \Delta (V_{ho}(t+\Delta) - V_{lo}(t) \\
+ 2\lambda\mu_{hn}(t+\Delta)\Delta(V_{ln}(t+\Delta) + P - V_{lo}(t) \\
+ V_{lo}(t+\Delta) - V_{lo}(t) + o(\Delta)
\end{aligned}
$$

3. divide by Δ and let $\Delta \to 0$ yields the result

Method 2: Express the value function in terms of Poisson processes and differentiate.

$$V_{ho}(t) = E_t\left[\int_t^{\tau} e^{-r(u-t)}du + e^{-r(\tau-t)}V_{lo}(\tau)\right]$$

where τ is the stopping time for switching to a low valuation.

Homework!

Hint: Use the Exponential Distribution for τ, then differentiate using Leibniz' rule, do algebra.

Nash bargaining

bargaining power of seller: $q \in [0, 1]$

$$\max_P (V_{ln} + P - V_{lo})^q (V_{ho} - P - V_{hn})^{1-q}$$

FOC:

$$P = (V_{lo} - V_{ln})(1 - q) + (V_{ho} - V_{hn})q$$

Equilibrium

Definition: An equilibrium is a value function for investors V_{ij}, a price P and a distribution of agents across states i and j such that

1. the price P is the outcome of Nash bargaining in a meeting where an assets gets sold
2. the value function V satisfies its system of differential equations
3. μ is a stationary distribution associated with the law of motions on μ_{ij}

We have just constructed one where there is trade at any meeting between an owner with low valuation and non-owner with high valuation.

In equilibrium:

$$P = \frac{1}{r} - \frac{\delta}{r} f(q, \lambda, \lambda_u, \lambda_d, s, r)$$

The second term is an illiquidity discount. Most importantly, as $\lambda \to \infty$ the discount disappears.

Trading friction λ directly influences liquidity measures.

Recall: average rate of trade meeting is $2\lambda\mu_{hn}$ for seller

Liquidity in equilibrium?

1. expected time to sell

$$\frac{1}{2\lambda\mu_{hn}} \times 250 \text{ trading days}$$

2. turnover ratio

$$\frac{2\lambda\mu_{hn}}{S} \times 250 \times 100 \text{ percent per year}$$