Lecture Notes

Ramsey Taxation

Thorsten V. Koeppl

Queen's University

Fall Term 2014

1 Taxes and Distortions

1.1 Model

Households:

$$\max_{\{c_t, n_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t) \tag{1.1}$$

subject to

$$(1+\tau_{ct})c_t + (1+\tau_{xt})x_t \leq (1-\tau_{kt})r_t k_t + (1-\tau_{nt})w_t n_t + T_t \tag{1.2}$$

$$k_{t+1} = (1-\delta)k_t + x_t \tag{1.3}$$

$$k_0 \text{ given} \tag{1.4}$$

$$c_t, k_{t+1} \geq 0, n_t \in [0,1] \tag{1.5}$$

Technology:

—▷ production function

$$y_t = F(k_t, n_t) \tag{1.6}$$

—▷ standard assumptions (e.g. Cobb-Douglass)

—▷ constant returns to scale implies zero profits in equilibrium

Government:

—▷ policy $\{z_t\}_{t=0}^{\infty} = \{g_t, \tau_{ct}, \tau_{xt}, \tau_{kt}, \tau_{nt}, T_t\}$

—▷ policy is feasible if it satisfies a flow budget constraint

$$g_t = \tau_{ct}c_t + \tau_{xt}x_t + \tau_{kt}r_t k_t + \tau_{nt}w_t n_t - T_t \tag{1.7}$$

—▷ households do not derive utility from government expenditure

1.2 Tax Wedges

Intratemporal Wedge

$$\frac{(1-\tau_{nt})}{(1+\tau_{ct})} = \frac{u_n(c_t, 1-n_t)}{u_c(c_t, 1-n_t)F_n(k_t, n_t)} \tag{1.8}$$

Intertemporal Wedge

$$\frac{u_c(c_t, 1-n_t)}{\beta u_c(c_{t+1}, 1-n_{t+1})} = \frac{(1+\tau_{ct})}{(1+\tau_{ct+1})}\left[(1-\delta)\frac{(1+\tau_{xt+1})}{(1+\tau_{xt})} + F_k(k_{t+1}, n_{t+1})\frac{(1-\tau_{kt+1})}{(1+\tau_{xt})}\right] \tag{1.9}$$

If there is no tax/subsidy on investment and taxes are constant, we obtain for the intertemporal wedge the much easier expression

$$\left[(1-\delta) + F_k(k_{t+1}, n_{t+1})(1-\tau_k)\right]. \tag{1.10}$$

1.3 Long-run Effects and Transitions

The steady state is given by the solution (c^{SS}, n^{SS}, k^{SS}) to

$$1 = \beta\left[(1-\delta) + \frac{(1-\tau_k)}{(1+\tau_x)}F_k(k^{SS}, n^{SS})\right] \tag{1.11}$$

$$\frac{u_c(c^{SS}, 1-n^{SS})}{u_n(c^{SS}, 1-n^{SS})} = \frac{(1-\tau_n)}{(1+\tau_c)}F_n(k^{SS}, n^{SS}) \tag{1.12}$$

$$g + c^{SS} + \delta k^{SS} = F(k^{SS}, n^{SS}) \tag{1.13}$$

—▷ Suppose $u(c, 1-n) = u(c)$, i.e. labour is inelastically supplied. Then, $\tau_c \neq 0$ does not influence the steady-state value of capital.

—▷ Why? Taxing consumption is non-distortionary if labour is inelastically supplied.

—▷ Hence, $\tau_x = \tau_k = 0$ is optimal and k^{SS} is first-best.

1.4 Transition

Dynamics are described by a second-order difference equation in capital k treating the exogenous variables – government policy as well as exogenous shocks – as given parameters

$$H(k_t, k_{t+1}, k_{t+2}; z_t, z_{t+1}) = 0 \tag{1.14}$$

One can use a shooting algorithm or DYNARE to compute transitions due to changes in tax policies.

2 Ramsey Taxation

2.1 Model

—▷ households as before

—▷ Government:

- expenditure $\{g(s^t)\}_{t=0}^{\infty}$ exogenously given
- chooses policy $\pi = \{\pi(s^t)\}_{t=0}^{\infty} = \{\tau_l(s^t), \tau_k(s^t), (1 + r_b(s^t))\}_{t=0}^{\infty}$
- flow budget constraint:

$$g(s^t) + (1 + r_b(s^t))b(s^{t-1}) = \tau_l(s^t)w(s^t)l(s^t) + \tau_k(s^t)[r(s^t) - \delta]k(s^{t-1}) + b(s^t) \tag{2.1}$$

- expenditure and choice of policy pins down the sequence of debt levels

2.2 Definition of Ramsey Equilibrium

- Allocation and Price rules: given policy π an allocation $x = (c, b, k, l)$ and prices (r, w) are realized in equilibrium
- $x(\pi) = \{x(s^t|\pi)\}_{t=0}^{\infty}$, $r(\pi) = \{r(s^t|\pi)\}_{t=0}^{\infty}$, $w(\pi) = \{w(s^t|\pi)\}_{t=0}^{\infty}$

- k_{-1} and b_{-1} are given (i.e. inelastically "supplied")
- hence: we take the tax rate and the interest rate on these variables as given

Definition 2.1. *A Ramsey Equilibrium for a given initial tax rate on capital $\tau_k(s_0)$ and a given initial interest rate $r_b(s_0)$ is a policy π and allocation and price rules (x, w, r) such that*

1. *π maximizes the household's utility subject to the government's flow budget constraint*
2. *for all policies π', $x(\pi')$ solves the household's problem taking π', $r(\pi')$ and $w(\pi')$ as given*
3. *for all policies π',*

$$w(s^t|\pi') = F_l(k(s^t|\pi'), n(s^t|\pi'), s^t)$$
$$r(s^t|\pi') = F_k(k(s^t|\pi'), n(s^t|\pi'), s^t)$$

   for all s^t, for all t.

—▷ The second and third condition impose that allocation and price rules form competitive equilibria.

—▷ The planner chooses then a *second-best* being restricted by the equilibrium choices of the agents.

Remark: We impose that households and firms behave optimally *for all* possible government policies. Suppose this was not the case. Then, take any policy and find an allocation and price rule that is a competitive equilibrium given this policy. The policy is optimal for an allocation rule that specifies that people supply zero labour for all policies except for the one policy we look at. Obviously, under appropriate assumptions on u, zero labour supply cannot be an equilibrium except for a 100% tax on labor income.

2.3 Ramsey Allocation Problem

2.3.1 Optimization Problem for the Planner

$$\max_{\{c(s^t),l(s^t),k(s^t)\}} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) u(c(s^t), 1 - l(s^t)) \tag{2.2}$$

subject to

$$c(s^t) + g(s^t) + k(s^t) = F(k(s^{t-1}), l(s^t), s^t) + (1 - \delta) k(s^{t-1}) \tag{2.3}$$

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) \left[u_c(s^t) c(s^t) + u_l(s^t) l(s^t) \right] = \tag{2.4}$$

$$u_c(s_0) \left[k_{-1} + (1 - \tau_{k0})(r(s^t) - \delta) k_{-1} + (1 + r_{b0}) b_{-1} \right] \tag{2.5}$$

—▷ The last condition is an "implementability" condition.

—▷ It is the household's intertemporal budget constraint...

—▷ ...and can be obtained by using the FONC to eliminate prices.

—▷ The government's budget constraint is implied by the resource constraint and the household's budget constraint.

2.3.2 Solution

Incorporate the implementability constraint into the objective function

$$W(c(s^t), l(s^t), \nu) = u(c(s^t), l(s^t)) + \nu \left[u_c(s^t) c(s^t) + u_l(s^t) l(s^t) \right] \tag{2.6}$$

Ramsey allocation problem in Lagrangian form:

$$\begin{aligned} \max_{\{c(s^t),l(s^t),k(s^t)\}} & \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) W(c(s^t), l(s^t), \nu) + \\ & - \nu u_c(s_0) \left[k_{-1}(1 + (1 - \tau_{k0})(r(s_0) - \delta) + b_{-1}(1 + r_{b0}) \right] \\ & + \nu(s^t) \left[F(k(s^{t-1}), l(s^t), s^t) + (1 - \delta) k(s^{t-1}) - c(s^t) + g(s^t) + k(s^t) \right] \end{aligned} \tag{2.7}$$

Two parts for the solution: a dynamic one and an initial one.

—▷ FONC for $t \geq 1$:

$$-\frac{W_l(s^t)}{W_c(s^t)} = F_l(s^t) \tag{2.8}$$

$$W_c(s^t) = \sum_{s_{t+1}|s^t} \beta\mu(s_{t+1}|s^t) W_c((s_{t+1}, s^t)) \left[1 - \delta + F_k((s_{t+1}, s^t))\right] \tag{2.9}$$

—▷ FONC for $t = 0$:

$$-\frac{W_l(s_0) - \nu\big(u_{cl}(s_0)\left[k_{-1}(1 + (1 - \tau_{k0})(r(s_0) - \delta) + b_{-1}(1 + r_{b0})\right] - u_c(s_0)[1 - \tau_{k0}]F_{kl}(s_0)\big)}{W_c(s_0) - \nu u_{cc}\big[k_{-1}(1 + (1 - \tau_{k0})(r(s_0) - \delta) + b_{-1}(1 + r_{b0})\big]} = F_l(s_0) \tag{2.10}$$

$$W_c(s_0) - \nu u_{cc}(s_0)\big[k_{-1}(1 + (1 - \tau_{k0})(r(s_0) - \delta) + b_{-1}(1 + r_{b0})\big] = \sum_{s_1|s_0} \beta\mu(s_1|s_0) W_c((s_1, s_0)) \left[1 - \delta + F_k((s_1, s_0))\right] \tag{2.11}$$

—▷ The above system of 4 equations describes the necessary conditions for a Ramsey equilibrium given by $(c(s^t), l(s^t), k(s^t))$ and ν. The Ramsey equilibrium allocation together with the multiplier ν has to satisfy these equations, the feasibility constraints and the implementability constraint.

—▷ To compute an equilibrium, take ν as fixed and solve the system of equations without the implementability constraint. Then check wether the implementability constraint is satisfied. If it is not, adjust the multiplier (or "price") ν accordingly.

—▷ Given the Ramsey Allocations, we can use the equilibrium conditions to easily recover prices and tax policies $(r, w, \tau_l, \tau_k, r_b)$.

2.4 Optimal Policies

1. Initial taxes are non-distortionary and, hence, optimal.

2. Tax Smoothing – Stokey and Lucas, Journal of Monetary Economics (1983)

3. Zero Capital Taxes in SS – Chamley, Econometrica (1986)

4. Chari, Christiano and Kehoe, Journal of Political Economy (1994)

   - Either capital taxes or interest rates can be state-independent (due to an indeterminancy of degree $N-1$ with N states).

   - The ex-ante tax rate on capital income is uniquely defined by

     $$\tau_k^e(s^t) = \frac{\sum_{s_{t+1}|s^t} q(s_{t+1}|s^t)\tau_k(s_{t+1}|s^t)\left[F_k(s_{t+1}|s^t)-\delta\right]}{\sum_{s_{t+1}|s^t} q(s_{t+1}|s^t)\left[F_k(s_{t+1}|s^t)-\delta\right]} \tag{2.12}$$

     where $q(s_{t+1}|s^t)$ is the Arrow-Debreu price of consumption in state $(s_{t+1}|s^t)$.

   - Consider the utility function,

     $$u(c,1-l) = \frac{c^{1-\sigma}}{1-\sigma} + V(1-l) \tag{2.13}$$

     where $\sigma > 0$ and V is strictly concave. Then optimal ex-ante average capital taxes are zero for all $t \geq 1$.

5. Optimal taxes are generally time-inconsistent.