Economics 817 **Winter 2017**

Advanced Macroeconomic Theory II **Thorsten Koeppl**

Assignment 5

(Due: Monday, April 3)

1. Consider the New Public Finance model with capital covered in the lecture notes. Assume that $T = 2$ and that there are only two idiosyncratic and two aggregate states each period (θ_L, θ_H) and (z_L, z_H), both occurring with equal probabilities. The planner can observe output and consumption, but not the idiosyncratic shocks θ_1 and θ_2.

   There is an initial capital stock k_0 and capital depreciates fully. Government expenditures are set to 0 and production is described by a Cobb-Douglas production function

$$F(K_t, Y_t) = z_t K_t^{\alpha} N_t^{1-\alpha}$$

   with $\alpha \in (0, 1)$. The per-period utility function is given by

$$\log c - \frac{1}{2}\left(\frac{y_t}{\theta_t}\right)^2$$

   where $\beta \in (0, 1)$.

   (a) Set up the perturbed problem and find the social discount factor $\lambda_2^* : (z_1, z_2) \to I\!R_+$. [Hint: Write out all equations for any possible perturbation given an aggregate state z_2.]

   (b) Find the capital tax scheme that implements the constrained efficient allocation.

   (c) Are all agents savings-constrained?

   (d) How would your answers change, if the idiosyncratic shock in $t = 1$ is perfectly persistent?

2. Consider the Pissarides model covered in the lecture notes. The planner's problem can be described by the value function

$$P(u) = \max_{\theta} ub + (1-u)y - k\theta u + \beta P(u_{+1})$$

subject to

$$u_{+1} = u + (1-u)\delta - um(1,\theta)$$

(a) Show that a solution to the Bellman equation is given by $P(u) = a_0 + a_1 u$ and find the constants a_0 and a_1.

(b) Show that this solution is unique.

(c) Suppose that the matching function is given by $M = v(1-e^{-u/v})$. How do changes in fixed costs k and in unemployment b affect market tightness in the efficient allocation and the distribution of surplus? Compare your results to the equilibrium conditions covered in the lecture notes.

3. Consider m sellers and n buyers. Suppose each buyer visits each seller with equal probability $\theta = 1/m$ like in an urn-ball experiment with replacement.

(a) What is the expected number of matches $M(n,m)$? [Hint: Find the probability for any seller that all buyers go elsewhere.]

(b) Suppose now that the ratio n/m stays constant. Show that the arrival rates $A_b = M(n,m)/n$ and $A_s = M(n,m)/m$ are decreasing in m.

(c) What happens if $m \to \infty$? Interpret your results in terms of the returns-to-scale of a matching function.