Economics 817 **Winter 2017**

Advanced Macroeconomic Theory II **Thorsten Koeppl**

Assignment 4

(Due: Friday, March 17)

1. Consider a CES aggregate consumption function given by

$$C = \left(\int c_i^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

Find the expression for $\lim_{\epsilon \to 1} C$ and for $\lim_{\epsilon \to \infty} C$. Interpret your answers.[Hint: Use L'Hôpital's rule.]

2. Consider the following neoclassical model. Preferences of the representative household are given by

$$E\left[\sum_{t=0}^{\infty} \beta^t \frac{\left(c_t^{\chi} n_t^{1-\chi}\right)^{1-\gamma}}{1-\gamma} \right].$$

The representative household owns all capital and invests each period into new capital x_t, supplies labor n_t and consumes c_t. It also faces a distorting tax on labor income that finances an exogenously given stream of government expenditures G. Hence, given a tax rate τ_{nt} the government budget constraint is given by

$$\tau_{nt}(s^t) w(s^t) n(s^t) = G_t(s^t).$$

The aggregate production technology given by the Cobb-Douglass production function

$$Y_t = z_t K_{t-1}^{\alpha} N_t^{1-\alpha}.$$

Assume throughout the question that $z_t = 1$.

   (a) Suppose that G and hence the tax rate is constant across time, i.e. $\tau_{nt} = \bar{\tau}_n$. Find the steady state for this economy when $\chi = 1$ and when $\chi = \frac{1}{2}$. How does it depend on $\bar{\tau}_n$?

(b) Let $\alpha = 0.33$, $\delta = 0.1$, $\beta = 0.9$ and $\gamma = 1$. Suppose that at $t = 0$ the economy is in steady state with a tax rate $\bar{\tau}_n = 0.2$, but the tax rate is permanently raised to $\bar{\tau}_n = 0.3$. Compute the transition to the new steady state for $\chi = 1$ and $\chi = \frac{1}{2}$. Graph all real variables (k, n, c) and the real interest rate r.

(c) Repeat part (b) assuming that the change in taxes is announced at $t = 0$, but takes effect only in period $t = 10$. Interpret the difference in your answers.

3. Consider the following economy. There is a representative household that can turn one unit of labour into one unit of output. His preferences are given by

$$E\left[\sum_{t=0}^{\infty} \beta^t u(c_t, 1 - l_t)\right].$$

A government finances an exogenously given stream of expenditures $\{g(s^t)\}_{t=0}^{\infty}$ by taxing labour income with tax rate $\tau_n(s^t)$ and by issuing state-contingent government debt denoted by $b(s^t)$.

(a) Formulate the Ramsey allocation problem.

(b) Suppose $b(s_0) = 0$ and $g(s_0) = G$. The government has to finance a war of unknown duration that every period consumes G resources. Specifically, assume that $P(g_{t+1} = G | g_t = G) = \alpha$ and $P(g_{t+1} = 0 | g_t = 0) = 1$. Characterize the optimal tax policy, i.e. labour tax rates and state-contingent government debt.