Economics 817 **Winter 2017**

Advanced Macroeconomic Theory II **Thorsten Koeppl**

Assignment 3

(Due: Monday, February 27)

1. Consider the following NK model. The Phillips Curve is given by

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa(y_t - y_t^n)$$

and the IS equation by the intertermporal Euler equation

$$y_t - y_t^n = -\frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - r_t^n) + E_t[y_{t+1} - y_{t+1}^n]$$

where

$$r_t^n = \rho + \sigma E_t[y_{t+1}^n - y_t^n]$$

is the natural rate of interest.

Consider first the policy rule $i_t = r_t^n$.

(a) Set up a state-space representation for this economy. [Hint: Eliminate i_t from the system.]

(b) Show that there exists a time path such that both inflation and the output gap are constant for any shock ϵ_t.

(c) Show that this time path is not unique.

Consider now the policy rule $i_t = \rho + \phi_\pi \pi_t$.

(a) Set up a state-space representation for this economy. [Hint: Eliminate i_t from the system.]

(b) Find a condition for which there is a unique stable solution for this economy.

(c) Characterize the solution $(\pi_t, y_t - y_t^n)$ by iterating forward on the system of difference equations.

2. Suppose now a central bank would like to minimize the loss function

$$L = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\alpha x_t^2 + \pi_t^2 \right) \right].$$

The central bank follows a simple rule given by $i_t = \rho + \phi_\pi \pi_t$.

The economy is described by a NK Phillips Curve and an IS equation as in the previous question. Assume that there are shocks to these equations given by

$$u_t \sim \mathcal{N}(0, \sigma_u)$$
$$\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon)$$

(a) Set up a state-space representation and solve for (π_t, x_t) as a function of the shocks (u_t, ϵ_t).

(b) Find the optimal policy ϕ_π^*. [Hint: Use the three equations directly in the objective function. Then, exploit that the shocks are normally distributed.

(c) Discuss how the parameters (α, κ) of the model and the shocks influence the optimal policy.

3. Consider a household with the following utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\epsilon_{ct} C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \right)$$

where ϵ_{ct} is a preference shock often referred to as a taste or demand shock. The household chooses aggregate consumption C_t, can save in nominal one-period bonds which have a price of Q_t, faces lump-sum transfers T_t and supplies labour N_t to firms for a nominal wage equal to W_t. Production in the economy is given by the production function

$$Y = A N_t^\alpha.$$

(a) Derive the Euler equation for the household.

(b) Log-linearize the Euler equation and express it in terms of the output gap $x_t = y_t - y_t^n$. How does it depend on ϵ_{ct}?

Consider a NK Philips Curve and a reaction function for monetary policy given by

$$\begin{aligned} \pi_t &= \beta E_t[\pi_{t+1}] + \kappa(y_t - y_t^n) + \epsilon_{\pi t} \\ i_t &= \bar{\imath} + \phi_\pi \pi_t + \phi_y (y_t - y_t^n) + \epsilon_{it}, \end{aligned}$$

where the natural level of output associated with flexible prices is given by

$$y_t^n = \psi a_t - \xi,$$

where $\psi = \frac{1+\eta}{\sigma\alpha+\eta+(1-\alpha)}$ and $\xi = \frac{\alpha \log \frac{\epsilon}{\alpha(\epsilon-1)}}{\sigma\alpha+\eta+(1-\alpha)}$. Set the policy parameters so that $\phi_\pi = 1.5$ and $\phi_y = 0.125$.

(c) Use DYNARE to estimate κ, ϵ and the parameters for the shocks $(\epsilon_{ct}, \epsilon_{at}, \epsilon_{it}, \epsilon_{\pi t})$ assuming that these shocks follow AR(1) processes with $\epsilon_t \sim \mathcal{N}(0, \sigma_{\epsilon_t})$. When doing so, calibrate all other parameters. [Hint: Calibrate all other parameters from an RBC model.]

(d) Compute impulse response functions for $(i_t, \pi_t, x_t, y_t, y_t^n)$ for each of the shocks.

(e) For the shock ϵ_{at}, how does varying the policy coefficients (ϕ_π, ϕ_y) change your results? Interpret your results.

(f) For the shock $\epsilon_{\pi t}$, how does varying the policy coefficients (ϕ_π, ϕ_y) change your results? Interpret your results.