Economics 817 **Winter 2017**

Advanced Macroeconomic Theory II **Thorsten Koeppl**

Assignment 2

(Due: Wednesday, February 1)

1. Consider a standard pure-exchange economy with infinite time horizon where time is discrete. There are two households $i = 1, 2$ with preferences represented by the utility function

$$\sum_{t=0}^{\infty} \beta_i^t \ln c_t^i,$$

where $0 < \beta_1 < \beta_2 < 1$. Both households face a constant endowment y over time.

   (a) Suppose people trade at $t = 0$ claims to consumption for every period. Define an Arrow-Debreu equilibirium for this economy.

   (b) Suppose households can borrow and lend every period t. Define a sequential equilibrium for this economy.

   (c) Find *all* Pareto-optimal Allocations, *all* Arrow-Debreu Equilibria and *all* Sequential Equilibria for this economy. Carefully explain your reasoning.

   (d) Find the minimum non-binding borrowing limit for the example you solved.

   (e) Find the "natural debt limit" $B_t^i = -\sum_{\tau=t}^{\infty} \Pi_t^{\tau} \frac{1}{1+r_\tau} y$, where r_t is the one-period interest rate. Is it binding for any equilibria?

   (f) Show that the Arrow-Debreu pricing functional is finitely additive, i.e. show that $\sum_{t=0}^{\infty} p_t < \infty$.

   (g) Finally, assume that people cannot borrow, i.e. $B_t^i = 0$ for all t and i. Find all Arrow-Debreu and Sequential Equilibria for this economy.

2. Consider the basic RBC model discussed in lecture. In every period, the utility function for households is given by

$$u(c, 1-n) = \frac{c^{1-\gamma}}{1-\gamma} + \theta \frac{(1-n)^{1-\eta}}{1-\eta}$$

with the household's endowment of time being normalized to 1. Future utility is discounted according to $\beta \in (0,1)$. The production function is given by

$$y = zk^{\alpha}n^{1-\alpha}$$

where $\ln z$ follows an AR(1) process with coefficient ρ and the innovations being normally distributed with zero mean and variance σ^2. Finally, assume that capital depreciates with rate δ.

(a) Calibrate your economy to data from the Canadian economy as discussed in class. [Hint: Do not calibrate, but set $\gamma = \eta = 1$.]

(b) Compute a steady state $(\bar{k}, \bar{c}, \bar{y})$. How well do the ratios c/y, k/y and x/y match the corresponding ratios in the data?

(c) Use `DYNARE` to compute impulse response functions for labour, consumption, capital and output. What correlations do you obtain between labour and output as well as between consumption and output?

(d) For what values of γ and η do you find impulse response functions for labour supply that do not match the sign one would expect from the data? [Hint: Consult King and Rebelo (1998) for the relevant business cycle facts.]

Consider now a model with variable capital utilization. The firm has a production function that is given by

$$y = z(\phi k)^{\alpha}n^{1-\alpha}$$

where $\phi \in (0,1)$ and depreciation is now given by

$$\delta(\phi) = A\phi^2.$$

(e) Assume that firms take as given the future level of capital. What is their revenue maximizing level of capital utilization ϕ^* if they take into account the increased cost of capital utilization in the form of higher depreciation?

(f) Use `DYNARE` to redo part c and interpret the differences you get. Make sure that you also plot the impulse response of ϕ to a technology shock and report its correlation with other jump variables. [Hint: You need to calibrate the level of A. To do so, match some measure of capital utilization relative to measured capital stock to $\bar{\phi}$ in steady state. Also, be careful to check that the solution of ϕ falls within the interval $[0, 1]$.]

3. Consider the following linear supply and demand equations

$$y^d_{t+1} = \alpha + \beta p_{t+1}$$

$$y^s_{t+1} = \gamma + \theta E_t p_{t+1}$$

Assume that expectations are formulated in adaptive fashion according to

$$E_t p_{t+1} = \phi p_t + (1 - \phi) E_{t-1} p_t$$

(a) Solve for the equilibrium price sequence. Under what conditions would prices be stable? [Hint: Use the lag-operator L on the price expectation. Note that the lag-operator applied to a constant is the constant itself, but with a variable we have $LX_t = X_{t-1}$.]

Assume now instead that inflation expectations are iid so that

$$E_t p_{t+1} = p_t + \epsilon_{t+1}$$

(b) Solve for the equilibrium price sequence.

(c) Derive the impulse response function of p_t in response to a one-time shock in inflation expectations.

(d) Under what conditions would prices be stable?