Economics 815 **Winter 2020**

Macroeconomic Theory **Thorsten Koeppl**

Assignment 4

(Due: Thursday, April 9)

1. Consider the following household utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\chi_t C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\nu}}{1+\nu} \right)$$

where χ_t is a preference shock often referred to as a taste or demand shock. Like in the benchmark NK model, the household chooses aggregate consumption C_t optimally over an index of individual goods, can save in nominal one-period bonds which have a price of Q_t, faces lump-sum taxes T_t and supplies labour N_t to firms for a nominal wage equal to W_t.

Production in the economy is given by the production function for individual goods

$$Y(i) = AN_t^{\alpha}.$$

There is also a government purchasing goods from firms. The total level of government consumption is given by G_t and is financed by lump-sum taxes T. Both the government and households choose their demand over individual goods as laid out in class.

(a) Derive the Euler-equation for the household and log-linearize it. [Hint: Use aggregate consumption.]

(b) Derive an IS equation in terms of a natural rate of interest taking into account that total aggregate demand is given by $Y_t = G_t + C_t$. [Hint: Log-linearize the market clearing condition around the steady state. To do so, define consumption shares $s_g = G/Y$ and $s_c = C/Y$. Then, use this equation in the log-linearized Euler equation for consumption.]

(c) Suppose there are no technology shocks. Set $i_t = \rho \equiv -\log \beta$. Show that an appropriately defined fiscal policy can perfectly stabilize the output gap and the inflation rate when χ_t changes over time, but is perfectly and contemporaneously observable by the government.

The model is now closed by the standard NK Philips Curve and a reaction function for monetary policy given by

$$\begin{aligned}\pi_t &= \beta E_t[\pi_{t+1}] + \kappa(y_t - y_t^n) \\ i_t &= \bar{\imath} + \phi_\pi \pi_t + \phi_y (y_t - y_t^n),\end{aligned}$$

where the natural level of output associated with flexible prices normalized to $y_t^n = 0$ so that the output gap is equal to actual output.

Consider the following AR(1) processes

$$\begin{aligned}\chi_t &= (1-\rho_\chi)\bar{\chi} + \rho_\chi \chi_{t-1} + \epsilon_{\chi t} \\ g_t &= (1-\rho_g)\bar{g} + \rho_g g_{t-1} + \epsilon_{gt}\end{aligned}$$

where $\rho_i \in (0,1)$ and ϵ_{it} are iid shocks that are independent of each other.

(d) Choose parameters for the shock processes. Justify your calibration.

(e) Use `DYNARE` to compute IRFs for a taste shock given the Taylor-type reaction function for monetary policy. Include $(i_t, \pi_t, r_t - r_t^n, y_t, y_t^n, x_t, c_t)$ and the shock χ_t in your output. [Hint: You can set $\bar{g}$ and $\bar{\chi} = 0$ for the program. Why?]

(f) Use `DYNARE` to compute IRFs for a government expenditure shock given the the Taylor-type reaction function for monetary policy. Include $(i_t, \pi_t, r_t - r_t^n, y_t, y_t^n, x_t, c_t)$ and your shock g_t in your output.

2. Consider the following NK model

$$\begin{aligned}\pi_t &= \beta E_t[\pi_{t+1}] + \kappa x_t + u_t \\ x_t &= E_t[x_{t+1}] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - \rho) + \epsilon_t \\ i_t &= \rho + \phi_\pi \pi_t.\end{aligned}$$

The two shocks – interpreted as supply and demand shocks – are iid and uncorrelated with variances given by σ_u^2 and σ_e^2 respectively. The long-run steady state values for the output gap and inflation are normalized to 0.

Iterating forward, the solution of this model is given by two linear difference equations

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \left(\frac{\sigma}{\sigma + \kappa \phi_\pi} \right) \begin{bmatrix} 1 & -\frac{\phi_\pi}{\sigma} \\ \kappa & 1 \end{bmatrix} \begin{bmatrix} \epsilon_t \\ u_t \end{bmatrix}.$$

Finally assume that the central bank has an objective function given by

$$L = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\alpha x_t^2 + \pi_t^2 \right) \right].$$

The central bank sets its reaction coefficient ϕ_π so as to minimize this loss function.

(a) Express the problem of the central bank in terms of only the variances σ_u^2 and σ_e^2. [Hint: You need to take as constraints the equilibrium processes for x_t and π_t.]

(b) Solve for the value of ϕ_π^* that minimizes the central bank's loss function.

(c) How does ϕ_π^* depend on the coefficient α, the variances of the shocks and κ? Interpret your results.

3. Consider a calibrated version of the New Keynesian Model from the previous question with r^n instead of ρ and a Taylor rule with coefficients (ϕ_π, ϕ_y).

   (a) Produce IRFs to (i) a permanent, negative technology shock and (ii) a permanent, positive cost push shock u_t. Interpret your results and show how they depend on (ϕ_π, ϕ_y). [Hint: Set the autocorrelation for the shock to 1 in `DYNARE`.]

   (b) Repeat your exercise assuming the information about the shock arrives in period 0, but the shocks only materialize in period 5. [Hint: Use `initval` and `endval` in `DYNARE` to look at a deterministic solution.]

   (c) (Optional) Consider a demand shock ϵ_t and a productivity shock that happen together. Assume a deterministic sequence for these shocks that you can associated with the impact of the CoVid-19 pandemic. Use the IRFs to give some advice for monetary policy.