Economics 815 **Winter 2020**

Macroeconomic Theory **Thorsten Koeppl**

Answer Key for Assignment 5

Answer to Question 1:

1. The household's problem is given by

$$\max_{c_t,m_t,b_t} E_0 \left[\sum_{t=1}^{\infty} \beta^t \left(\frac{1}{1-\sigma} \left[c_t \phi \left(\frac{m_t}{c_t} \right) \right]^{1-\sigma} - 1 \right) \right]$$

subject to

$$P_t c_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + w_t n_t - T_t$$

m_0 and b_0 given

Define total asset $A_t = B_t + M_t$, the budget constraint can be written as

$$P_t c_t + Q_t A_t + (1 - Q_t) P_t m_t \leq A_{t-1} + w_t n_t - T_t$$

where $m_t = M_t / P_t$ denotes real balances.

Setting up the Lagrangian

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} \left[c_t \phi \left(\frac{m_t}{c_t} \right) \right]^{1-\sigma} - 1 \right) + \sum_{t=0}^{\infty} \lambda_t \left(A_{t-1} + w_t n_t - T_t - P_t c_t - Q_t A_t - (1 - Q_t) P_t \right) \right]$$

yields the FOC

$$\lambda_t P_t = \beta^t c_t^{-\sigma} \left[\phi \left(\frac{m_t}{c_t} \right) \right]^{-\sigma} \left[\phi \left(\frac{m_t}{c_t} \right) - \phi' \left(\frac{m_t}{c_t} \right) \frac{m_t}{c_t} \right]$$

$$\lambda_t P_t (1 - Q_t) = \beta^t c_t^{-\sigma} \left[\phi \left(\frac{m_t}{c_t} \right) \right]^{-\sigma} \phi' \left(\frac{m_t}{c_t} \right)$$

$$\lambda_t Q_t = E_t[\lambda_{t+1}]$$

The first equation pins down the optimal choice of c_t, the second the choice of real balances m_t and the last one is just the Euler equation that determines the real interest rates. All are expressed as a function of prices (Q_t, P_t).

2. To find the steady state, we first look at market clearing and normalize $A = 1$ and the endowment of labour to $n = 1$. Then,

$$C = Y = AN^{\alpha} = 1$$

and the wage rate is given by $w = \alpha$.

Next, we use the first-order condition with respect to real balances and consumption to obtain

$$\frac{\phi'\left(\frac{m_t}{c_t}\right)}{\phi\left(\frac{m_t}{c_t}\right) - \phi'\left(\frac{m_t}{c_t}\right)\frac{m_t}{c_t}} = 1 - Q_t = \frac{i_t}{1+i_t}$$

In steady state, all variables are constant and since $c = 1$ gives

$$\frac{\phi'(m)}{\phi(m) - \phi'(m)m} = \frac{i}{1+i} = 1 - Q \tag{0.1}$$

Define now $\phi\left(\frac{m_t}{c_t}\right) - \phi'\left(\frac{m_t}{c_t}\right)\frac{m_t}{c_t} = \Delta_t$. For the Euler equation, we have

$$\frac{Q_t}{P_t}\left(\beta^t c_t^{-\sigma}\left[\phi\left(\frac{m_t}{c_t}\right)\right]^{-\sigma}\right)\Delta_t = E_t\left[\frac{1}{P_{t+1}}\left(\beta^{t+1} c_{t+1}^{-\sigma}\left[\phi\left(\frac{m_{t+1}}{c_{t+1}}\right)\right]^{-\sigma}\right)\Delta_{t+1}\right]$$

which collapses in steady state to

$$\frac{Q_t}{P_t} = \frac{\beta}{P_{t+1}}$$

since all real variables are constant. Hence, the Euler equation pins down the real interest rate as

$$\frac{1+i_t}{1+\pi_t} = \frac{1}{\beta}$$

Note that this establishes a relationship between the nominal interest rate and the inflation rate, since $1/\beta$ is constant. There are no real effects of inflation, because preferences are separable with respect to real balances and consumption. To see this, just transform the utility function by adding the constant $(1-\sigma)$ and using the log-operator.

Since real balances are constant, the money supply and prices must grow at the same rate. Finally, we have one degree of freedom here which is here inflation. We regard this as an exogenous policy variable.

Remark: Take M_0 and P_0 as given. Policy decides on a constant nominal money growth rate $(1+\mu)$. The government obtains *seignorage* from money growth equal to

$$\frac{M_t - M_{t-1}}{P_t} = \left(\frac{\mu}{1+\mu}\right)\frac{M_t}{P_t}$$

This real(!) revenue – aka the inflation tax – is transfered back to households.

Use the budget constraint and set $B_t = 0$ for simplicity. Dividing by P_t, we obtain

$$c_t + \frac{M_t}{P_t} = \frac{M_{t-1}}{P_t} + \frac{w_t}{P_t} - \frac{T_t}{P_t}$$

Real transfers are from (i) profits and (ii) seignorage. Hence, the budget constraint collapses again to $y = c = 1$ which we have used already in the form of market clearing.

3. Substituting $\phi(m) = \frac{m}{m+B}$ into the steady condition, we obtain

$$m = \sqrt{\frac{B}{1-Q}} = \sqrt{B\left(\frac{1+i}{i}\right)}$$

Thus, the utility function in steady state with the normalization $c = 1$ and any interest rate i is given by

$$u(1,m) = \frac{1}{1-\sigma}\left[\left(\frac{1}{1+\sqrt{B\frac{i}{1+i}}}\right)^{1-\sigma} - 1\right]$$

Utility is thus maximized when nominal interest rates are zero ($i = 0$). This implies that $m^* = \infty$ and $u_{max} = 0$.

Remark: This is simply the Friedman rule. Note that for this to be consistent with steady state, prices must decrease at the rate of time preference β.

Remark: Note that this model is dichotomous. The real side is independent of nominal variables. Thus, having consumption normalized to $c = 1$ is independent of any nominal interest rate i. When nominal interest rates increase, people want to economize on their real balances which reduces utility.

4. We need to find $c(i)$ such that for any given interest rate i utility is given by

$$\frac{1}{1-\sigma}\Big[\left[(1+c(i))\,\phi\left(m(i)\right)\right]^{(1-\sigma)}-1\Big]=u(1,m^*)=0.$$

This is equivalent to

$$1+c(i)=\frac{1}{\phi\left(m(i)\right)}.$$

Using $m=\sqrt{\frac{B(1+i)}{i}}$ and $\phi=\frac{1}{1+\frac{B}{m}}$ we get

$$c(i)=\sqrt{B\left(\frac{i}{1+i}\right)}$$

Remark: This is an important idea. To be able to determine the welfare loss from inflation, we calculate by how much the household needs to be compensated in terms of average consumption to obtain the same utility as in a steady state with the Friedman rule which defines the efficient equilibrium. Since we have normalized consumption in steady state for $i=0$ to 1, we can view $c(i)$ as the % of additional steady state consumption the household requires.

5. I am using the FRED database of the Federal Reserve Bank of St. Louis. I have found an average of 10.62% for the time period 1980 - 1993 and 4.42% for the time period start of 1994 to the end of 2007. The interest rate used is the overnight rate, a *short-term* interest rate.

Using the values, we obtain for the welfare loss of inflation

$$c(0.1061)=0.0139$$
$$c(0.0442)=0.009$$

Hence, the welfare gain from introducing the inflation targeting regime is 1.30% of annual consumption, since we have used annual frequency when calculating the welfare loss. To put this into dollar terms, total private consumption expenditure at the end of

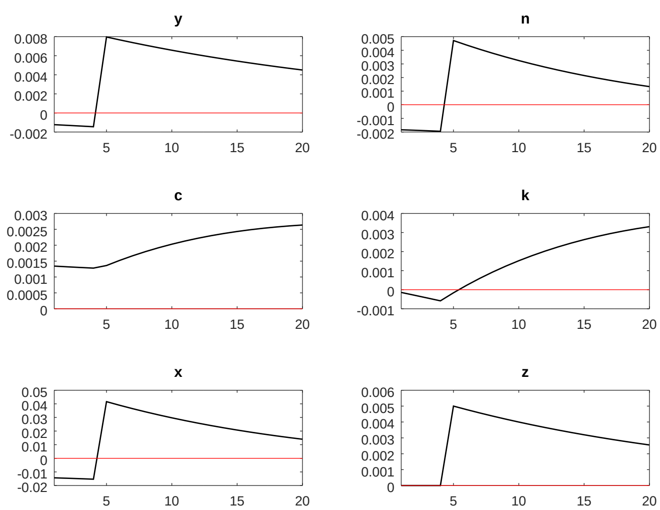
2014 was roughly \$1,100bn. Hence, one can argue that annually the inflation targeting regime saves people \$14.3bn relative to the pre targeting regime.

Remark: This figure crucially depends on what we assume for the parameter B.

Answer to Question 2:

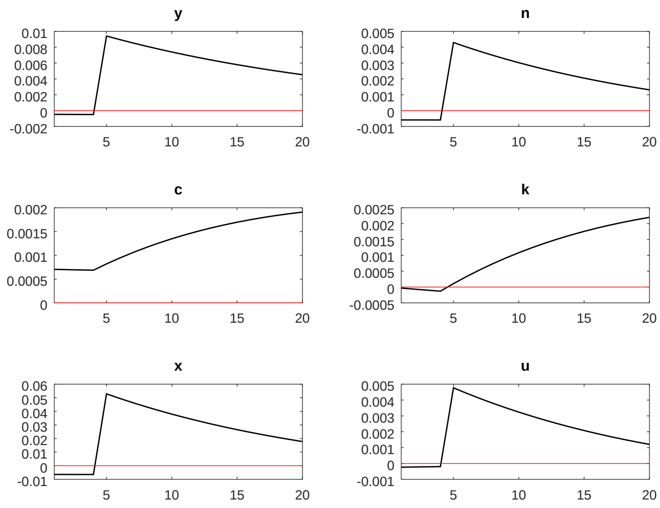
1. The figure below shows the reaction to a positive news shock about future, increased productivity. We report the impulse response functions only for the the news shock and only for the first 20 periods. The behavior after the shock is identical to the standard RBC model.

   Surprisingly, the labour supply and investment fall which is not what one would expect. After all, this is good news! Why?



The labour demand curve does not shift at all since productivity does not increase before period 4. There is however a positive income effect as future expected income has increased. The consumer smoothes utility by consuming some of the capital stock and increasing leisure. Hence, the labour supply curve shifts and we have lower equilibirum employment. Once the productivity has increased, the dynamics are identical to a regular productivity shock.

2. For completeness, below are the impulse responses in the model with capital utilization. Nothing changes, with capital utilization being reduced which complements the reduction in investment.



Remark: This is clearly a puzzle. We would expect that good news about productivity would set off an investment bonanza! What can change this? Try some stuff. Habit formation? The New Keynesian model? What else?

Answer to Question 3:

1. I chose parameters like the ones in Gali. They are as follows

   - $\theta = 0.66$ – prices on average reset every three quarters
   - $\sigma = \eta = 1$ – log utility and quadratic costs for labour
   - $\beta = 0.99$ – about 4% per cent risk-free rate
   - $\epsilon = 6$ – from literature
   - $\alpha = 2/3$ – standard benchmark

The parameters for the AR(1) process can be taken from the Solow residual estimation in the first half. I have assumed here $\rho_a = 0.9$ and have used a 1% deviation from steady state as a shock.

Remark: Calibrating parameters should be done with care in general. However, there is no standard way of carrying this out. Of course, this means that the key parameters like θ or ϵ involve a lot of judgement. A value of $\theta = 0.66$ implies that the expected time for a firm to reset prices is given by $\sum_{t=0}^{\infty} \left(\frac{2}{3}\right)^n \frac{n}{2} = 3$ or three quarters. There are many micro studies that try to see how often prices are in fact reset.

Remark: The parameter ϵ expresses the strategic complementarity in the price setting of firms. It is hard to calibrate. Therefore, one usually resorts to picking a reasonable value and then conducts a sensitivity analysis. For ϵ large, products are highly substitutable. Hence, for any given degree of price stickiness, a small change in the price of a firm that is allowed to change its prices will attract a lot of additional demand. This implies that such firms will change their price by only a small amount, but that there is a relatively large reallocation of demand. For $\epsilon \to \infty$ we have perfect competition, so that there is strong complementarity in price setting (aka firms deviate only little in their price setting). To the contrary, when $\epsilon \to 1$, firms need to change their prices a lot to attract additional demand.

2. For the impulse response function associated with these parameters, see the slides for Lecture XVIII.

3. For θ close to 1 (larger price stickiness), the output gap becomes larger as firms cannot adjust their prices. In other words, the shock has a large impact on real output, but only a lower impact on inflation. As a consequence, monetary policy (aka nominal interest rates) is less able given a coefficient ϕ_π to stabilize inflation and the output gap in response to a technology shock.

4. For ϵ close to 1, the output gap is unaffected. However, κ increases. Hence, inflation responds more strongly to any fixed change in the output gap. The incentive to change prices aggressively has increased for firms that are able to do so, since monopoly power is larger. This implies that monetary policy is more effective for any given coefficient ϕ_{π}.