Economics 815 **Winter 2020**

Macroeconomic Theory **Thorsten Koeppl**

Assignment 5

(Due: Tuesday, March 31 – in class)

1. Consider a basic model without labour-leisure choice and capital. There are two assets, nominal bonds and money. Households have a per-period utility function given by

$$u(c,m) = \frac{1}{1-\sigma}\left\{[c\phi\,(m/c)]^{1-\sigma} - 1\right\},$$

where $m_t = \frac{M_t}{P_t}$ denote real balances. The function $\phi(m/c)$ will be specified further below.

Furthermore, denote nominal bond holdings by B_t and the nominal interest rate by i_t. The household supplies one unit of labour inelastically and earns a wage w_t for doing so. The production function is given by

$$Y_t = AN_t^{\alpha}.$$

(a) Set up the household's problem and derive the first-order conditions with respect to consumption, savings and real balances.

(b) Find the steady state and normalize the economy so that in steady state $c = 1$. [Hint: Since labour is inelastically supplied, market clearing always implies that $Y_t = C_t = A$. This implies that we can normalize the economy to have $c_t = C_t = 1$ in any (!) equilibrium.]

(c) Using this normalization, suppose $\phi(m) = \frac{1}{1+\frac{B}{m}}$. Find the value of m^* for which utility is maximized in steady state? What is the level of utility $u(1, m^*)$ associated with this steady state?

(d) Find the additional *consumption certainty equivalent* that is required so that the household is as well off when facing nominal interest rates i as in the normalized optimal steady state, i.e., find $\gamma(i)$ so that $u(1 + \gamma(i), \phi(m(i))) = u(1, m^*)$. [Hint: You need to find m as a function of i first.]

(e) Calculate the welfare gains from introducing the inflation targeting regime in Canada from 1994 to 2008 relative to the period 1980 to 1994 as a percentage of steady state consumption. To do so, assume $B = 0.002$ and estimate long-run nominal interest rates from the data for the two time periods.

2. Consider the basic RBC model for Assignment 3 and the RBC model with variable capital utilization from Assignment 4.

   In period 0, there is now news of a negative productivity shock that will arrive in 4 quarters from now. Note that this shock can be modelled as

$$z_t = \rho z_{t-1} + \epsilon_{t-4}$$

   Hence, when people learn about this shock in period t, they anticipate that productivty will change in period $t+4$ by the amount ϵ. After that period, productivity will return according to an AR(1) process back to its original, long-run level.

   (a) Use `DYNARE` to plot impulse response functions for your calibrated economies. [Hint: In `DYNARE`, one can just lag the exogenous shock directly by using `e(-4)`.]

   (b) How do the responses compare to a regular negative productivity shock.

3. Consider the New Keynesian model described by the equations

$$\begin{aligned}
\pi_t &= \beta E_t[\pi_{t+1}] + \kappa(y_t - y_t^n) \\
y_t - y_t^n &= E_t[y_{t+1} - y_{t+1}^n] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - r_t^n) \\
i_t &= \rho + \phi_\pi \pi_t + \phi_y (y_t - y_t^n).
\end{aligned}$$

   Set the policy parameters so that $\phi_\pi = 1.5$ and $\phi_y = 0.125$. Assume that productivity follows an AR(1) process given by

$$a_t = \rho_a a_{t-1} + \epsilon_{at}.$$

For the parameters of the productivity process (ρ_a, σ_a) use the values that you have estimated before.

The natural level of output associated with flexible prices, but monopolistic competition is given by

$$y_t^n = \psi a_t - \xi,$$

where $\psi = \frac{1+\nu}{\sigma\alpha+\nu+(1-\alpha)}$ and $\xi = \frac{\alpha \log \frac{\epsilon}{\alpha(\epsilon-1)}}{\sigma\alpha+\nu+(1-\alpha)}$ and α is the parameter of the production function.

This implies that the natural rate of interest is given by

$$r_t^n = \rho + \sigma\psi E_t[a_{t+1} - a_t]$$

The parameter κ is proportional to the factor

$$\frac{(1-\theta)(1-\beta\theta)}{\theta}\frac{\alpha}{\alpha+\epsilon(1-\alpha)}$$

where ϵ is the price elasticity of demand and θ is the probability at which individual firms update their prices.

(a) Choose parameters $(\alpha, \kappa, \epsilon, \beta, \rho, \sigma, \nu)$ in the Canadian context and justify their values.

(b) Compute IRFs for $i_t, r_t, r_t^n, \pi_t, y_t, y_t^n$ and $x_t = y_t - y_t^n$ in `DYNARE` for a technology shock. Interpret your results. [Hint: The real interest rate is defined as in the lecture slides.]

(c) Now set θ close to 0. How do your IRFs change? Interpret your results.

(d) Now set ϵ close to 1. How do your IRFs change? Interpret your results.