Economics 815 **Winter 2020**

Macroeconomic Theory **Thorsten Koeppl**

Answer Key for Assignment 4

Answer to Question 1:

1. As per lecture 9, all variables in the model should be positively correlated with TFP and output. This is consistent with the experience of the aggregates in business cycles, historical episodes, and structural empirical estimates. Note that increasing the intertemporal elasticity of substitution *uniformly* across consumption and leisure (e.g. $\gamma = \eta = 5$) will yield a counterintuitive negative correlation between labour in response to a positive technology shock. This is due to the strong income effect relative to the substitution effect. Try it.

2. A common criticism against the standard RBC framework is that it relies on an unreasonable degree of intertemporal substitution of leisure. In the data, changes in hours worked are large relative to productivity fluctuations, in contrast with the general RBC model. Setting $\gamma = 5$ and $\eta = 0$ increases the intertemporal elasticity of consumption and makes utility linear in labour which corresponds to Hansen (JME, 1985) indivisible labor model. With indivisible labor, the aggregate willingness to substitute leisure intertemporally is extremely high. Hence, with indivisible labor, the model exhibits larger fluctuations in hours worked relative to productivity. Interestingly, this also implies that θ will have no influence on the cyclical properties of the model when solved using log-linerization methods.

Answer to Question 2:

1. The household's problem is given by

$$\max_{c_t, x_t, n_t} E_0 \sum_{t=1}^{\infty} \beta_t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + \frac{(1-n)^{1-\nu}}{1-\nu} \right]$$

subject to

$$c_t + x_t = w_t n_t + r_t k_t \quad \text{for all } t \text{ and } z_t$$

$$k_{t+1} = x_t + (1-\delta) k_t$$

$$k_0 \text{ and } z_0 \text{ given}$$

where z_t is the stochastic value of productivity. One can then set up the Lagrangian

$$L = E_0 \Big[\sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + \frac{(1-n)^{1-\nu}}{1-\nu} \right] + \sum_{t=0}^{\infty} \lambda_t \left[w_t n_t + r_t k_t + (1-\delta) k_t - k_{t+1} - c_t \right] \Big]$$

and derive the FOC

$$\lambda_t = \beta^t c_t^{-\sigma}$$

$$\lambda_t w_t = \beta^t (1-n_t)^{-\nu}$$

$$\lambda_t = E_t \left[\lambda_{t+1} \left[r_{t+1} + 1 - \delta \right] \right]$$

$$c_t + k_{t+1} = w_t n_t + r_t k_t + (1-\delta) k_t$$

which express the labour-leisure choice, the Euler equation and the budget constraint. Solving we obtain

$$\frac{c_t^{-\sigma}}{(1-n_t)^{-\nu}} = \frac{1}{w_t}$$

and

$$1 = \beta E_t \Big[\left(\frac{c_t}{c_{t+1}} \right)^{\sigma} (r_{t+1} + 1 - \delta) \Big]$$

The firm's problem is entirely standard and we have

$$\max_{k_t, n_t} z_t C_t^{\omega} k_t^{\alpha} n_t^{1-\alpha} - r_t k_t - w_t n_t$$

which gives the following first-order condition

$$r_t = \alpha z_t C_t^{\omega} k_t^{\alpha-1} n_t^{1-\alpha} = \alpha \frac{y_t}{k_t}$$
$$w_t = (1-\alpha) z_t C_t^{\omega} k_t^{\alpha} n_t^{-\alpha} = (1-\alpha)\frac{y_t}{n_t}$$

Importantly, both the firm and the household takes aggregate consumption as given. This implies that the individual decision makers do not consider how their choice influence aggregate ouptut.

2. To derive the steady state equilibrium, we solve first for the equilibrium in terms of output y. In a second step, we then determine the output level y.

   Importantly, we set throughout $c = C$, but take the first-order conditions as derived in part (a) of the question.[1]

   First Step:

   In steady state, the Euler equation is given by

$$1 = \beta(r + 1 - \delta)$$

   where

$$r = \alpha \frac{y}{k}.$$

   Hence, we have for the capital output ratio

$$\frac{k}{y} = \frac{\alpha\beta}{1-\beta(1-\delta)}.$$

   Market clearing is given by $C + \delta k = y$ so that

$$\frac{C}{y} = 1 - \delta\left[\frac{\alpha\beta}{1-\beta(1-\delta)}\right]$$

   and labour input being given by

$$\frac{(1-n)^{\nu}}{C^{\sigma}} = \frac{1}{(1-\alpha)\frac{y}{n}}$$

[1] If we had for example (a measure of) two households, we would have to adjust this condition to be $2c = C$.

or

$$\frac{(1-n)^{\nu}}{n} = \frac{(\frac{C}{y})^{\sigma} y^{\sigma-1}}{1-\alpha}.$$

So far, we have expressed the steady state (k, n, C) as a function of output y.

<u>Second Step:</u>

We can now rewrite the production function to obtain

$$\begin{aligned} y &= zC^{\omega}k^{\alpha}n^{1-\alpha} \\ &= z\left(\frac{C}{y}\right)^{\omega} y^{\omega+\alpha} \left(\frac{k}{y}\right)^{\alpha} n^{1-\alpha} \\ &= y^{\omega+\alpha} z \left(1 - \delta\left[\frac{\alpha\beta}{1-\beta(1-\delta)}\right]\right)^{\omega} \left(\frac{\alpha\beta}{1-\beta(1-\delta)}\right)^{\alpha} n^{1-\alpha} \end{aligned}$$

Hence, we have two equations

$$y = \left[z\left(1 - \delta\left[\frac{\alpha\beta}{1-\beta(1-\delta)}\right]\right)^{\omega} \left(\frac{\alpha\beta}{1-\beta(1-\delta)}\right)^{\alpha} n^{1-\alpha}\right]^{\frac{1}{1-\omega-\alpha}}$$

and

$$y = \left(\frac{(1-n)^{\nu}}{n} \frac{1-\alpha}{\left(1-\delta\left[\frac{\alpha\beta}{1-\beta(1-\delta)}\right]\right)^{\sigma}}\right)^{\frac{1}{\sigma-1}}.$$

in (y, n) which we could solve numerically. We can then recover values for C, k and prices from the other equations.

<u>Remark 1:</u> We need another restriction on parameters. For the question to make sense we need that the production function is increasing in labour input. This is only the case if

$$w + \alpha < 1.$$

<u>Remark 2:</u> Note that the steady state production function exhibits increasing returns to scale when expressed in terms of labour input since

$$\frac{1-\alpha}{1-w-\alpha} > 1.$$

Remark 3: One also needs to show that there exists at least one solution that is different from $n = 0$. The first equation is increasing in n starting from 0. The second equation is increasing in n if and only if $\sigma < 1$. We need to distinguish two cases.

For $\sigma < 1$, we have that the second function starts out at $y = 0$, but diverges for $n \to 1$. Consequently, for there to be a steady state, the first function needs to increase fast enough.

For $\sigma > 1$, we have that the second function diverges for $y = 0$ as n approaches 0 and it goes to 0 as $n \to 1$. Hence, in this case there always must be a steady state.

The case where $\sigma = 1$ (and $\nu = 1$) will be considered below.

3. The social planner takes into account that higher consumption increases output. As output increases, so does income which can thus be used to finance increased consumption. In other words, there is a demand externality where higher demand (or consumption) can finance "itself" by increasing output and, thus, income.

   The social planner solves the following maximization problem

$$\max_{c_t,k_{t+1},n_t} E_0 \sum_{t=1}^{\infty} \beta_t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + \frac{(1-n)^{1-\nu}}{1-\nu} \right]$$

subject to

$$c_t = C_t$$

$$C_t + k_{t+1} = y_t + (1-\delta)k_t \quad \text{for all } t \text{ and } z_t$$

$$y_t = z_t C_t^{\omega} k_t^{\alpha} n_t^{1-\alpha}$$

$$k_0 \text{ and } z_0 \text{ given}$$

   Note that the planner takes into account now that aggregate consumption influences output. Furthermore, he directly chooses output, consumption and capital taking as given the feasibility constraint. To the contrary, the household took prices – and, hence, his income as given – when choosing consumption. The Lagrangian is given by

$$L = E_0 \Big[\sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + \frac{(1-n)^{1-\nu}}{1-\nu} \right] + \sum_{t=0}^{\infty} \lambda_t \left[z_t c_t^{\omega} k_t^{\alpha} n_t^{1-\alpha} + (1-\delta)k_t - k_{t+1} - c_t \right] \Big]$$

which gives the following FOC

$$\beta^t C_t^{\sigma} = \lambda_t \left(1 - \omega \frac{y_t}{C_t}\right)$$
$$\beta^t (1-n_t)^{-\nu} = \lambda_t (1-\alpha)\frac{y_t}{n_t}$$
$$\lambda_t = E_t\Big[\lambda_{t+1}\left(\alpha\frac{y_{t+1}}{k_{t+1}} + 1 - \delta\right)\Big]$$
$$C_t + k_{t+1} = y_t + (1-\delta)k_t$$

where we have directly expressed everything in terms of aggregate consumption. Hence, the only equation that changes is the optimal consumption decision. This of course, will influence the Euler equation and, thus, capital accumulation and, therefore, also output and labour input.

For the optimal steady state, we still have

$$\frac{k^*}{y^*} = \frac{\alpha\beta}{1-\beta(1-\delta)}$$
$$\frac{C^*}{y^*} = 1 - \delta\left[\frac{\alpha\beta}{1-\beta(1-\delta)}\right].$$

This implies that the capital-output and consumption-output ratios are still constant and the same as in the competitive equilibrium. Also, how output is related to labour input is given by the same relationship as in the equilibrium (see part (b) above).

However, the labour-leisure choice has changed and is now given by

$$\frac{C^{-\sigma}}{(1-n)^{-\nu}} = \frac{1-\omega\frac{y}{C}}{(1-\alpha)\frac{y}{n}}$$

which can be rearranged to give

$$\frac{(1-n^*)^{\nu}}{n^*} = \frac{(\frac{C^*}{y^*})^{\sigma}(1-\omega\frac{y^*}{C^*})y^{*(\sigma-1)}}{1-\alpha}$$

This implies that for any value of C/y, the right-hand side is lower. Consequently, n has to increase which implies that output is larger in the optimal steady state than in the equilibrium one. This is due to the demand externality. To conclude, while consumption and capital are constant fractions of total output, the efficient level of consumption, capital and output are higher than the ones in competitive equilibrium.

4. The tax has to be designed in such a fashion as to equate the equilibrium steady state output with the efficient one. This can be achieved by equating the intratemporal Euler equations.

   Suppose then that the household has to pay a tax τ_c for each unit of consumption. The household's budget constraint then becomes

$$c_t(1+\tau_t) + x_t = w_t n_t + r_t k_t$$

   In steady state with $\sigma = \nu = 1$, the intra-temporal Euler equation is given by

$$\frac{1-n}{n} = \frac{(1+\tau_c)}{(1-\alpha)} \frac{C}{y} \tag{0.1}$$

   Recall that at optimal allocation, we have

$$\frac{1-n^*}{n^*} = \frac{(1-\omega\frac{y^*}{C^*})}{(1-\alpha)} \frac{C^*}{y^*} \tag{0.2}$$

   Comparing these two equations, we see that for $\tau_c = -\omega\frac{y^*}{C^*}$ we can achieve the efficient allocation. This implies that individual consumption should be subsidized with the subsidy being equal to the positive externality that individual consumption has on output. Why? The right-hand side of this last expression is just the derivative of output with respect to aggregate consumption. From part (b), we thus have

$$\tau_c = \omega\left(1-\delta\left[\frac{\alpha\beta}{1-\beta(1-\delta)}\right]\right)^{-1}.$$

5. The two figures compare the standard RBC model where $\omega = 0$ and the model with a demand externality where $\omega = 0.5$.

   One can make three important observations about the results. First, the shock is amplified by the demand externality. Output, consumption and investment all increase significantly. Labor input stays rougly constant upon impact – a fact of log utility –, but remains higher with the demand externality. Second, the impact of the shock is more persistent. Hence, the demand externality causes the shock to be felt longer in the economy. Third, the shape of output is interesting. Output peaks after about 10 periods implying a hump-shaped response not present in the RBC model.

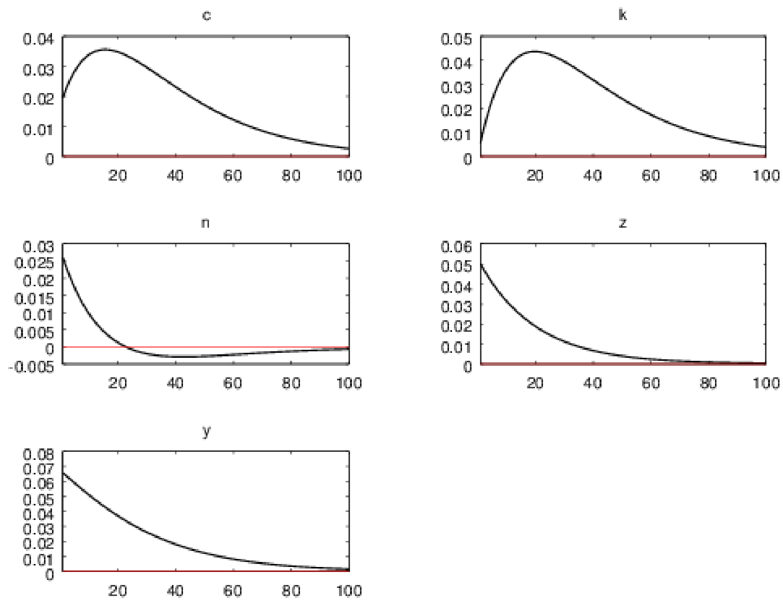


Figure 1: $\omega = 0$

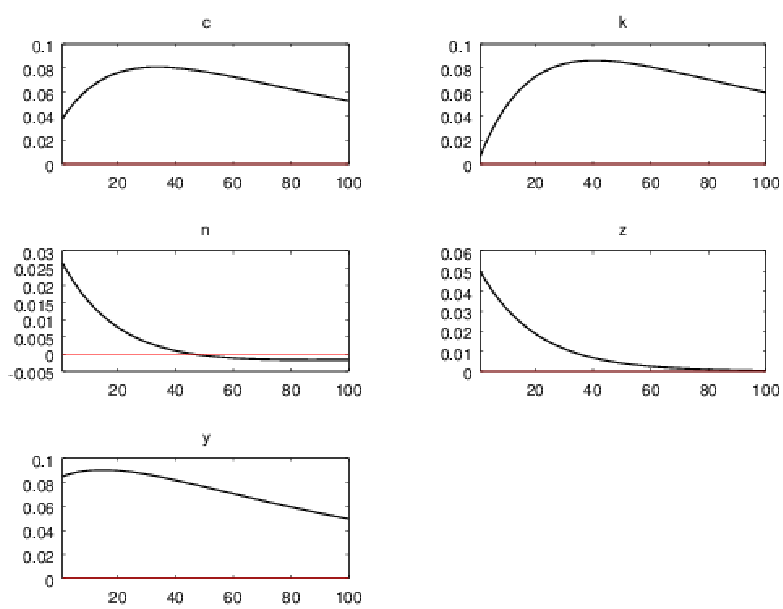


Figure 2: $\omega = 0.5$

Answer to Question 3:

1. To find the optimal rate of capital utilization, we set up a social planner's problem. Below, we will compare it to a decentralized problem. The problem is given by

$$\max_{c_t,k_{t+1},n_t,u_t} = E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\log c_t + \frac{(1-n_t)^{1-\nu}}{1-\nu}\right)\right]$$

subject to

$$\begin{aligned}
&c_t + x_t = z_t(u_t k_t)^{\alpha} n_t^{1-\alpha} \quad \forall t \text{ and } z_t\\
&k_{t+1} = x_t + (1-\delta(u_t))k_t \quad \forall t \text{ and } z_t\\
&\delta(u_t) = \delta u_t^{\theta}\\
&k_0 \text{ and } z_0 \text{ given}
\end{aligned}$$

Plugging investments and depreciation into the market clearing conditions, we obtain for the FOC

$$\begin{aligned}
&\frac{\beta^t}{c_t} = \lambda_t\\
&\frac{\beta^t}{(1-n_t)^{\nu}} = \lambda_t(1-\alpha)\frac{y_t}{n_t}\\
&\alpha\frac{y_t}{u_t} = \delta\theta u_t^{\theta-1}k_t\\
&-\lambda_t + E_t\left[\lambda_{t+1}(1-\delta u_{t+1}^{\theta} + \alpha\frac{y_{t+1}}{k_{t+1}})\right] = 0
\end{aligned}$$

Solving these equations, the optimal solution is given by

$$\begin{aligned}
&\frac{c_t}{(1-n_t)^{\nu}} = (1-\alpha)\frac{y_t}{n_t}\\
&u_t = \left(\frac{\alpha}{\delta\theta}\frac{y_t}{k_t}\right)^{\frac{1}{\theta}}\\
&1 = \beta E_t\left[\frac{c_t}{c_{t+1}}(1-\delta u_{t+1}^{\theta} + \alpha\frac{y_{t+1}}{k_{t+1}})\right]\\
&c_t + k_{t+1} = (1-\delta u_t^{\theta})k_t + y_t
\end{aligned}$$

where we have used feasiblity.

2. The production function with the optimal rate of capital utilization is given by

$$\begin{aligned} y_t &= z_t(u_t k_t)^\alpha n_t^{1-\alpha} \\ &= z_t \left(\left(\frac{\alpha}{\theta\delta} z_t k_t^{\alpha-1} n_t^{1-\alpha} \right)^{\frac{1}{\theta-\alpha}} k_t \right)^\alpha n_t^{1-\alpha} \\ &= \left(\frac{\alpha}{\theta\delta} \right)^{\frac{\alpha}{\theta-\alpha}} z_t^{\frac{\theta}{\theta-\alpha}} k_t^{\frac{\alpha(\theta-1)}{\theta-\alpha}} n_t^{\frac{\theta(1-\alpha)}{\theta-\alpha}} . \end{aligned}$$

   There are a few important insights. First, we should impose that $\theta > 1$. This is reasonable, since depreciation is a cost and, hence, it should increase convexly with utilitzation.

   Second, the production function is still CRS. This implies that the utilization of capital has no direct influence on the returns from scaling the input factors.

   Third, the productivity paramters has an exponent that is larger than 1. Hence, productivity shocks will be amplified directly.

3. Let's have a look at the FOC for capital utilization again. It is given by

$$u^{\alpha-1}\alpha z k^\alpha n^{1-\alpha} = \delta\theta u^{\theta-1}$$

   Holding n fixed, the LHS (RHS) is the marginal benefit (cost) of utilizing capital. For any value $u_t < 1$, the marginal costs are larger than the marginal benefits. Starting out at $u = 1$ and decreasing capital utilization would increase output only if n increases. But this cannot be optimal as the marginal product of labour falls with capital utilization. Consequently, it would be optimal to set $u = 1$.

   Remark: Admittedly, this was a silly question to ask. We should have $\theta \geq 1$ and any sensible calibration implies that $\alpha < 1$.

4. From the FOCs, our steady state equations are

$$1 = \beta\left(1 - \delta u^{\theta} + \alpha\frac{y}{k}\right)$$
$$\frac{c}{(1-n)^{\nu}} = (1-\alpha)\frac{y}{n}$$
$$u = \left(\frac{\alpha}{\theta\delta}\frac{y}{k}\right)^{1/\theta}$$
$$c = y - \delta u^{\theta} k$$
$$y = z(uk)^{\alpha} n^{1-\alpha}$$

We have 5 equations that solve for the four unknowns (c, k, n, u) and output y. Interestingly, the Euler equation is almost identical to our earlier one, since

$$1 = \beta\left(1 + \left(\frac{\theta - 1}{\theta}\right)\alpha\frac{y}{k}\right)$$

5. <u>Problems:</u>

1) `DYNARE` clearly has trouble finding a steady state for $\theta \leq 1$. The reason is that the cost function is not strictly convex.

2) We cannot calibrate the model appropriately unless there is some $B > 1$ for weighing leisure in the utility function. The reason is that in steady state

$$\frac{c}{y} = (1-\alpha)\frac{1-n}{n}$$

which puts limits on the value of labour n which we think is usually around 20-30% of time.

For the calibration, use standard values for (α, β) and the AR(1) parameters. The preference parameters are set to 1 anyway. For δ, we choose a smaller value of 0.01. This is by inspection of the steady state to get reasonable values for the steady state values of capital utilization. The values below show these values in logs.

Variable	Steady State Value
y	0.420139
c	0.239816
n	-1.10794
k	3.82172
x	-1.38169
u	-0.299114

The model steady state gives us a capital utilization of about 75%. This is a compromise as the equilibrium depreciation rate per year of less than 4% is clearly too low. We would think about 10% to be appropriate. Also, the capital utilization rate is seems too low. Finally, the capital/output ratio is way off at about 30.

Note that I have calibrated $B = 20$! I have done this to get a reasonable value for n to be close to 0.3 in steady state. In general, you may have trouble to get $n \in [0, 1]$ if you do not set an appropriate value for B. Note that B has no influence on the IRFs.

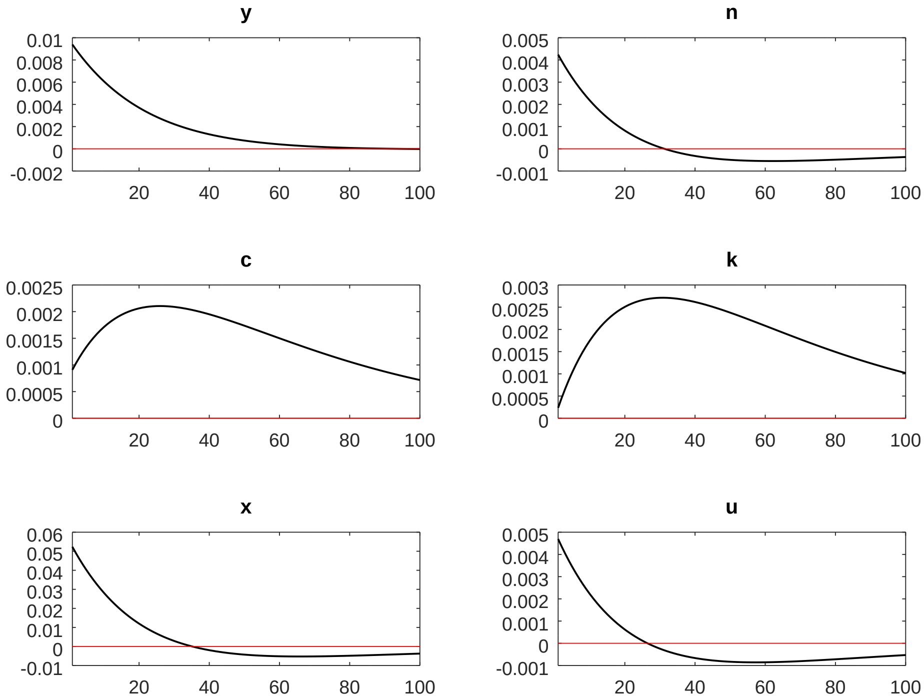
6. The IRFs are qualitatively the same as in the RBC model. In response to a positive productivity shock, we have that both, investment and capital utilization increase. Hence, the economy reacts on an intensive margin (using existing capital more) and an extensive margin (investing into more capital). Eventually, the economy will still reduce existing capital and convert it into consumption.

   The effects of the technology shock should be amplified through three channels. First, we know from above that productivity shocks are amplified in this model realtive to the RBC model. We have

$$\frac{\theta}{\theta - \alpha} = 1.2$$

   for our calibration. Second, there is an additional, immediate effect through higher capital utilization which can react immediately to the productivity shock. Third, the elasticity of output to labour has changed since the exponent on labour is given by

$$\hat{\alpha} = \frac{\theta(1 - \alpha)}{\theta - \alpha} = 0.8$$



This is the elasticity of output with respect to changes in labour input. As capital utilization increase, so does the marginal product of labour. This is captured by the increased coefficient $\hat{\alpha}$. Consequently, output increases much stronger given the increase in labour. This allows both consumption and investment increase more, too.

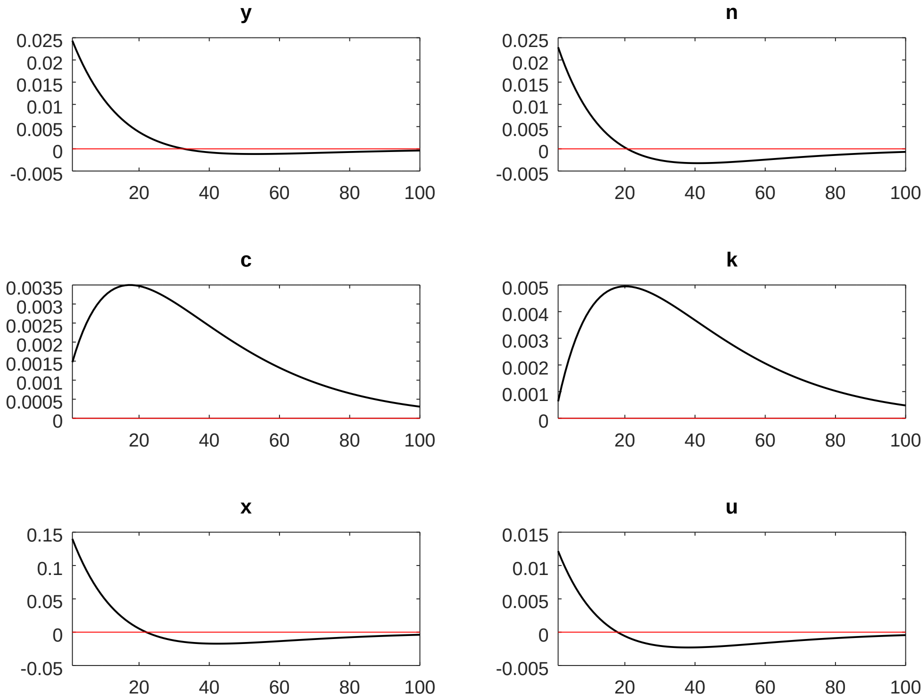
7. With indivisible labour, the intratemporal Euler equation becomes

$$B = \frac{(1-\alpha)}{c_t} z_t (u_t k_t)^{\alpha} n_t^{-\alpha} = (1-\alpha)\frac{y_t}{c_t n_t}$$

Given that $\frac{c}{y}$ is estimated to be 0.75, $\alpha = 0.33$, and we want the model to replicate the hours worked to be equal to the long run average in the data – $n = 1/3$, from steady state labour-leisure choice equation, we have

$$\begin{aligned} B &= (1-\alpha)\frac{y}{cn} \\ &= \frac{1-0.33}{0.75} \times 3 = 2.4 \end{aligned}$$

With linear labour, the parameter B does not matter for the impulse response function. However, it influences the steady state. Below are the results for our calibration



Variable	Steady State Value
y	0.432459
c	0.252136
n	-1.09562
k	3.83404
x	-1.36935
u	-0.299114

The figure clearly shows that the reaction of labour supply is much stronger, about 4 times as strong. Consequently, the output response is also larger, but only by a factor of 2 due to less amplification through capital utilization.

Remark: Let's look a decentralized version of this economy where households own the capital, but only submit a fraction u_t to firms that fully use this capital. Hence, households indirectly determine utilization and depreciation of the capital stock.

The constraint for the household is given by

$$c_t + k_{t+1} - (1 - \delta(u_t))k_t \leq w_t n_t + r_t(u_t k_t)$$

The firm maximizes its profit with

$$\max_{k_t, n_t} z_t \tilde{k}_t^{\alpha} n_t^{1-\alpha} - r_t \tilde{k}_t - w_t n_t$$

where $\tilde{k}_t = u_t k_t$ in equilibrium. The interpretation is that the household rents out u_t per cent of its capital stock.

The FOCs yield

$$r_t = \frac{\partial \delta}{\partial u_t} = \delta \theta u_t^{\theta - 1}$$
$$r_t = \alpha \frac{y_t}{u_t k_t}$$

which is identical to the planner's problem.

<u>Remark:</u> Let's look an economy where household submit all their capital to firms, but the firms decide upon how much of the capital to utilize. We assume that the rental of capital is at a rental rate r_t and a compensation for the (variable) capital utilization.

The households budget constraint is given by

$$c_t + k_{t+1} - k_t = (r_t + \delta(u_t))k_t + w_t n_t$$

The firm now chooses also the utilization rate u_t. This can be thought of renting all machines, but driving them only at u_t%.

$$\max_{k_t, n_t, u_t} z_t \tilde{k}_t^{\alpha} n_t^{1-\alpha} - r_t \tilde{k}_t - w_t n_t - \delta(u_t) k_t$$

The FOC for the firm yields

$$r_t + \delta(u_t) = \alpha \frac{y_t}{k_t}$$
$$\frac{\partial \delta}{\partial u_t} = \delta \theta u_t^{\theta - 1} k_t = \alpha \frac{y_t}{u_t}$$

which gives the identical choice for u as in the planner's problem.

Finally, the payment to households is given by

$$(r_t + \delta(u_t)) = \alpha \frac{y_t}{k_t} k_t = \alpha y_t$$

as before.

Answer to Question 4:

1. Define the vector

$$z_t = (\tau_t, g_t, y_t)$$

where τ_t is real tax revenue, g_t is real government expenditure and y_t is real GDP. The structural VAR can be transformed in a reduced form VAR by premultiplying the regression equation with $\mathbf{B_0}^{-1}$ to obtain

$$\begin{aligned} z_t &= \mathbf{B_0}^{-1}\mathbf{B_1} z_{t-1} + \cdots + \mathbf{B_0}^{-1}\mathbf{B_p} z_{t-p} + \mathbf{B_0}^{-1} w_t \\ &= \mathbf{A_1} z_{t-1} + \cdots + \mathbf{A_p} z_{t-p} + u_t. \end{aligned}$$

2. The first step is to take nominal values for τ_t and g_t and detrend them with a GDP deflator to obtain real values. One can work directly with real GDP data. The data will not be stationary. Hence, one should at least detrend the variables by taking log differences to obtain a VAR in growth rates.

The second step is to determine the lag length p from the data. There are various criteria available.

The third step is to estimate the parameter matrices $\mathbf{A_i}$ by OLS.

Remark: A complication arises from the fact that the three time series are cointegrated. Hence, one should think about correcting for this cointegration relationship before running a VAR analysis. Ideally, this would be done in the form of a VECM.

3. In general, we can put restrictions to identify the VAR according to

$$\mathbf{A} u_t = \mathbf{B} w_t$$

The question asks you to put restrictions on $\mathbf{A} = \mathbf{B}_0$ which is often called the "A model". Since we do not normalize the original covariance matrix of the residuals in the VAR, we now need to put 1 on the diagonals and find three more restrictions.

In general, one can put restrictions on both matrices ("AB model"). I find it more convenient to restrict the matrix $\mathbf{B} = \mathbf{B_0}^{-1}$ in a particular way. This is often called the "B model" and is closer to a recursive identification. One can use the normalized covariance matrix for the shocks and, hence, one needs only the three restrictions for identification. Of course, all models are related and can be mapped into each other.

<u>Identifying restrictions:</u>

1) Current tax revenue and expenditure do not influence each other.

2) The elasticity of tax revenue to GDP is given by 1.16 (see OECD 2015). This means that a 1% increase in GDP increases taxes by 1.16% contemporaneously.

<u>Remark:</u> The first restriction assumes that a shock to tax policy or to expenditures do not contemporaneously influence the other variable. This seems reasonable since I strip out interest payments from government expenditures. Also, we allow for any contemporaneous influence of shocks to GDP to these variables and vice versa. This seems like a benign assumption.

This yields the following matrix to be estimated for the structural VAR

$$\mathbf{B_0}^{-1} = \begin{bmatrix} \cdot & 0 & 1.16 \\ 0 & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

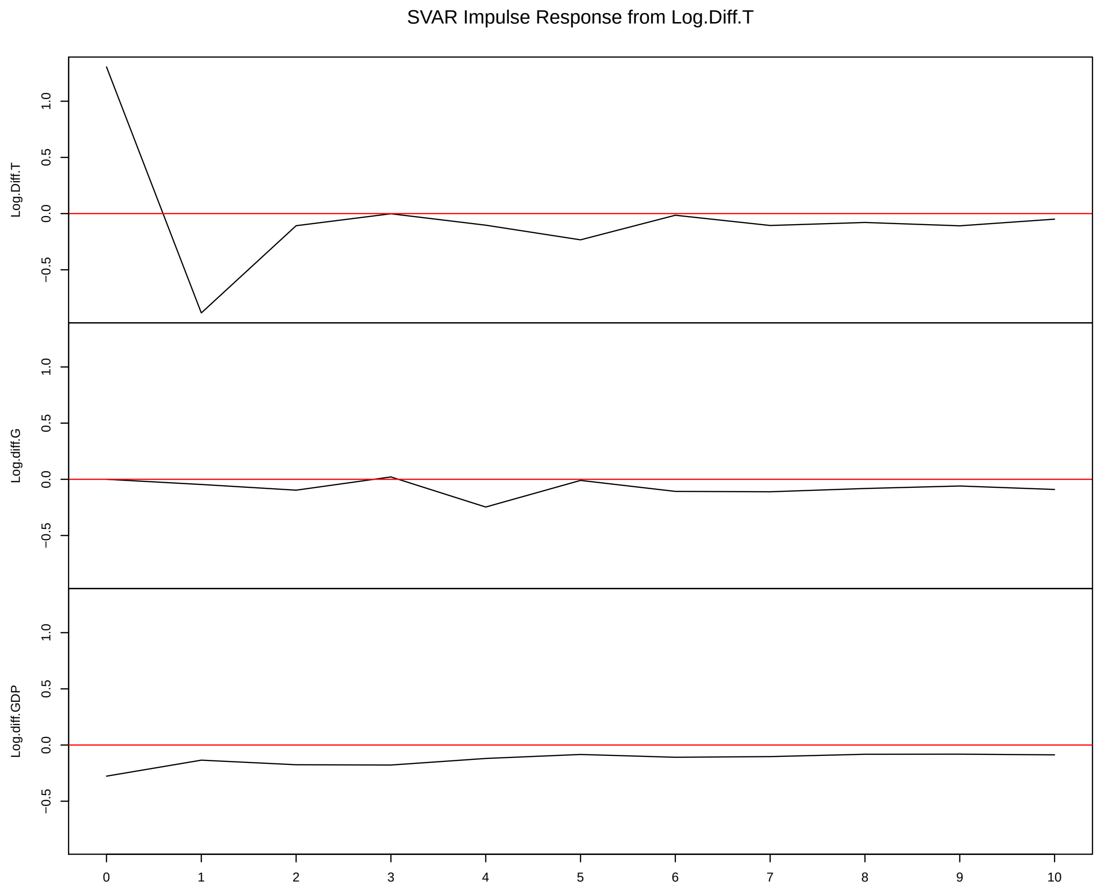
4. We can now use this matrix to figure out the structural IRFs. They are given by

$$\mathbf{B_0}^{-1} w_t$$

for the first period and then by the $\mathbf{A_i}$ matrices and the values for z_{t-i} for the future periods.

The estimated matrix is given by

$$\mathbf{B_0}^{-1} = \begin{bmatrix} 1.31 & 0 & 1.16 \\ 0 & 1.37 & 0.19 \\ -0.27 & -0.18 & 0.6 \end{bmatrix}$$

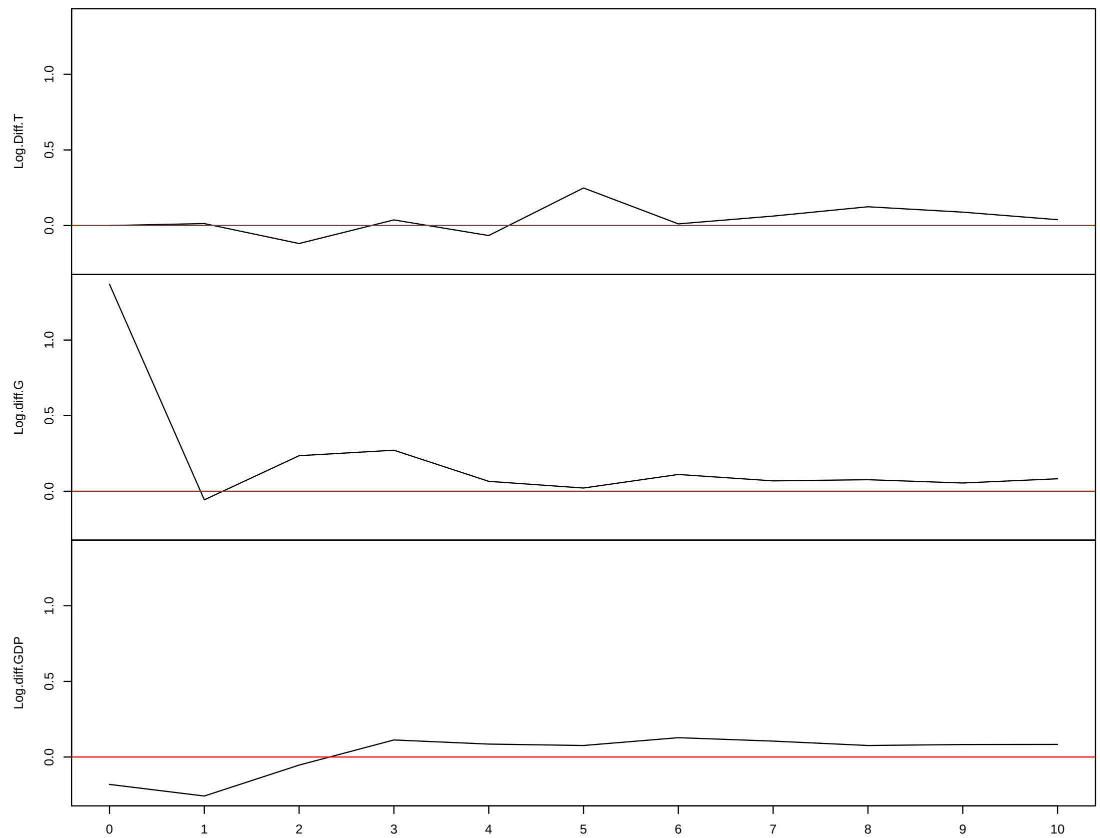


5. I first plot the IRF to a one standard deviation shock in the growth rate of tax revenue. There is a negative impact on GDP and a lagged impact on expenditures.

   Next, I look at a shock to expenditures. Interestingly, there is a negative impact on GDP first, but in the medium run, this turns into a stimulus effect after about three quarters. Tax revenue remains flat until it increases after 5 quarters.

   Finally, I look at a GDP shock. Tax revenue increases (but this is kind of assumed in the model unless expenditures fall dramatically) and expenditure falls after one quarter, but then increase as well. There seems to be a persistent effect on tax revenue for quite some time.

SVAR Impulse Response from Log.diff.G



SVAR Impulse Response from Log.diff.GDP