Economics 815 **Winter 2020**

Macroeconomic Theory **Thorsten Koeppl**

Assignment 4

(Due: Tuesday, March 10 – in class)

1. Finish the last question from Assignment 3.

2. Consider a standard RBC model with households having preferences given by the per-period utility function

$$u(c,n) = \frac{c^{1-\sigma}}{1-\sigma} + \frac{(1-n)^{1-\nu}}{1-\nu}.$$

The production function is given by

$$Y_t = z_t C_t^{\omega} K_t^{\alpha} N_t^{1-\alpha}$$

where $\omega \in (0,1)$ and C_t denotes *aggregate* consumption and is given by

$$C_t = \int c_{i,t} di$$

where i is an individual household and the total measure of households is normalized to 1. Assume that technology evolves according to an AR(1) process and that capital depreciates at rate δ.

(a) Set up the household's and the firm's problem and find the first-order conditions. [Hint: Households and firms take aggregate consumption as given.]

(b) Find the steady state competitive equilibrium for this economy. [Hint: When choosing individual consumption, the household is not taking into account that aggregate consumption positively influences output.]

(c) Find the optimal (i.e., welfare maximizing) steady state in this economy. Interpret the difference to the equilibrium steady state. [Hint: A planner maximizes utility subject to the constraint of facing the production function.]

(d) Set $\sigma = \nu = 1$. Find a tax or subsidy on consumption so that the competitive equilibrium is given by the optimal steady state.

Computational Part

(e) Use your calibration for the Canadian economy and set $\omega = 0.5$. Using `DYNARE`, find the impulse response functions for (Y, C, K, N) to a technology shock z_t. Compare your impulse response functions to the case $\omega = 0$ and interpret the differences.

3. Consider again the standard RBC model where preferences are given by

$$u(c, n) = \log c + \frac{(1-n)^{1-\nu}}{1-\nu}.$$

The production decision involves now also a decision about the utilization of capital. More specifically, output is given by

$$Y_t = z_t \left(u_t K_t\right)^{\alpha} N_t^{1-\alpha}$$

where $u_t \in [0, 1]$ is the rate at which capital is utilized.

Assume further that the rate of capital utilization is costly as it influences the law of motion of capital according to

$$K_{t+1} = (1 - \delta_t) K_t + X_t$$
$$\delta_t = \delta u_t^{\theta}$$

(a) Derive the FOC describing the optimal rate of capital utilization.

(b) Suppose that $\theta > \alpha$ and derive the production function with the optimal rate of capital utilization.

(c) What happens if $\theta \leq \alpha$?

(d) Find the steady state for this economy.

Computational Part

(e) Suppose $\theta = 0.6$. Use a calibrated version of the RBC model to derive the IRFs for (y, c, n, k, u, x) in response to a technology shock.

(f) Compare your IRFs with the IRFs of the standard RBC model. [Hint: For $\theta = 0$, we have $\delta_t = \delta$ and $u_t = 1$.] Interpret your results in terms of amplification and persistence of the shock.

(g) Repeat the exercise assuming that labour is indivisible so that the utility function is now given by

$$u(c, n) = \log c + B(1 - n)$$

Note that the parameter B needs to be calibrated appropriately. Interpret your results in terms of amplication and persistence of the shock.

4. Consider the following VAR model for real taxes τ, real government expenditure g and real output y_t given by

$$B_0 z_t = B_1 z_{t-1} + \ldots B_p z_{t-p} + w_t.$$

Assume that all variables are detrended for example by using log-difference.

(a) Transform the model into a reduced form VAR.

(b) Use quarterly Canadian data to estimate the reduced form VAR on the vector (τ_t, g_t, y_t). When doing so, determine the optimal lag structure from the data.

(c) Set up an identification strategy for the VAR by putting restrictions only on the matrix B_0. [Hint: Normalize the diagonal elements of this matrix to 1.]

(d) Estimate a structural VAR based on your identification strategy. Show your impulse response functions for a shock to each of the three variables.

(e) Interpret your results. In particular, what do we learn about the short-run and the long-run impact of fiscal policy on output?