Economics 815 **Winter 2020**

Macroeconomic Theory **Thorsten Koeppl**

Assignment 3

(Due: Tuesday, February 18 – Main Office or by e-mail)

1. Consider the model of income inequality discussed in lecture. Talent x is exponentially distributed with cdf given by $1 - e^{-\delta x}$. Income y increases exponentially with talent according to

$$y = e^{-\mu x}$$

   (a) Derive an expression for the share of income that goes to the top n% of the population.

   (b) Draw a Lorenz curve and calculate the Gini coefficient as a function of the coefficient for Pareto inequality.

   (c) Find data on the Gini coefficient in Canada for 1990, 2000 and 2010. What does the model tell us about how the distribution of income and talent has changed over time?

2. Consider a production function that is given by

$$Y_t = AK_t^{\alpha} \left(N_t H_t\right)^{1-\alpha}$$

where H_t is the level of human capital in period t. Assume that $N_t = N_0 = 1$. Accumulation of physical and human capital is given by

$$K_{t+1} = X_{kt} + (1 - \delta)K_t$$

$$H_{t+1} = X_{ht} + (1 - \delta)H_t$$

with K_0 and H_0 given. Assume that investments (X_{kt}, X_{ht}) are financed by current output Y_t and, hence, not available for consumption. Finally, utility of representative households is given by

$$u(c_t) = \frac{c^{1-\gamma}}{1-\gamma}$$

with $\gamma > 0$ and discounting between periods according to $\beta \in (0, 1)$.

(a) Derive the intertemporal Euler equation for a planner in this economy.

(b) Find the growth rate in the balanced growth path where consumption, output and both forms of capital grow at a common growth rate g.

(c) For the balanced growth path, show that the economy is isomorphic to an economy with a production function given by $Y_t = \tilde{A}K_t$.

(d) What is the savings rate of the economy in the balanced growth path?

3. Consider an OLG economy where each generation has utility given by

$$(1-\beta)\log c_1 + \beta \log c_2$$

where $\beta \in (0, 1)$. People inelastically supply one unit of labour and build up capital when young.

Production is described by a Cobb-Douglass production function given by

$$Y = AK^{\alpha}N^{1-\alpha}$$

Capital fully depreciates every period.

Consider first a social planner that chooses c_1, c_2, k directly to maximize the utility of a representative generation.

(a) Set up the social planning problem. [Hint: Feasibility requires to aggregate consumption by the young and old generation alive in a period.]

(b) Find the optimal steady state allocation.

Consider now a competitive economy where generations earn a wage w_1 and invest k_1 into capital for a return r_2 when old.

(c) Derive the Euler equation for a representative generation.

(d) Find the steady state equilibrium. [Hint: Market clearing requires to aggregate consumption by the young and old generation alive in a period.]

(e) For which parameters does the equilibrium coincide with the optimal allocation in steady state?

4. Consider the basic RBC model discussed in lecture. In every period, the utility function for households is given by

$$u(c, 1-n) = \frac{c^{1-\gamma}}{1-\gamma} + \theta \frac{(1-n)^{1-\eta}}{1-\eta}$$

with the household's endowment of time being normalized to 1. Future utility is discounted according to $\beta \in (0,1)$. The production function is given by

$$y = zk^{\alpha}n^{1-\alpha}$$

where $\log z$ follows an AR(1) process with coefficient ρ and the innovations being normally distributed with zero mean and variance σ^2. Finally, assume that capital depreciates with rate δ.

(a) Derive the steady state conditions in terms of (k, c, y).

(b) Calibrate your economy to data from the Canadian economy as discussed in class. In particular, pick θ such as to match a target for the fraction of time spent working. Carefully describe your strategy for calibration and the data that you are using. [Hint: Do not calibrate, but set $\gamma = \eta = 1$.]

Computational Part

(c) Compute steady state values for (k, c, y) and θ. How well do the ratios c/y, k/y and x/y match the corresponding ratios in the data?

(d) Use `DYNARE` to compute impulse response functions for labour, consumption, capital and output. What correlations do you obtain between labour and output as well as between consumption and output?

(e) Adjust your parameters to $\gamma = 5$, $\eta = 0$ and recalibrate θ to the same target. Report how your answers to part (d) change and interpret these changes.