Economics 815 **Winter 2020**

Macroeconomic Theory **Thorsten Koeppl**

Assignment 2

(Due: Tuesday, February 4 – in class)

1. Consider the following economy. The economy lasts for two periods. A representative household has an endowment of a single good equal to $y = 1$ in the first period, but no endowment in the second period. Preferences over consumption are given by

$$u(c_1, c_2) = \frac{c_1^{1-\gamma}}{1-\gamma} + \beta \frac{c_2^{1-\gamma}}{1-\gamma}$$

where $\gamma > 0$ and $\beta \in (0, 1)$. The household can invest into capital in period 1 to obtain a deterministic gross return of r in period 2.

The capital is being used by a representative firm to produce output in the second period according to the production function

$$f(k) = k^{\alpha}$$

where $\alpha \in (0, 1)$. Once capital has been used, it fully depreciates. The firm is owned by the household who receives all the firm's profits after it has payed for the rental of capital.

(a) Set up the household's decision problem with a sequence of budget constraints.

(b) Set up the firm's decision problem.

(c) Define a competitive equilibrium for this economy.

(d) Derive a single equation that describes the competitive equilibrium in terms of capital k invested in the first period and the parameters of the model (α, β, γ).

Computational part

(e) Set $\alpha = 0.3$, $\beta = 0.9$ and $\gamma = 2$. Solve numerically for the equilibrium capital stock k, consumption c_1 and c_2 and the equilibrium interest rate r.

(f) Produce graphs for the equilibrium capital stock, the interest rate and consumption as a function of the parameter $\gamma \in (0, 10)$. [WARNING: $\gamma = 1$ is a special case. Which one?]

(g) Provide economic intuition for the resulting graph.

2. In each period, a household has a stochastic endowment $\underline{y}$ or $\bar{y}$. Uncertainty is described by the following Markov transition matrix

$$\Pi = \begin{bmatrix} 0.8 & 0.2 \\ \mu & (1-\mu) \end{bmatrix}$$

where the first (second) row gives the probabilities to go from $\underline{y}$ ($\bar{y}$) today to a new state tomorrow.

(a) Assume that the state today is given by $\underline{y}$. What is the probability distribution across states tomorrow?

(b) Find the long-run, stationary distribution across the two states $\underline{y}$ and $\bar{y}$. How does it depend on the parameter μ?

3. Consider the following economy with uncertainty. Households have preferences that are given by

$$u(c_1, c_2) = E_0 \left[\frac{c_0^{1-\gamma}}{1-\gamma} + \beta \frac{c_1^{1-\gamma}}{1-\gamma} + \beta^2 \frac{c_2^{1-\gamma}}{1-\gamma} \right]$$

where $\gamma > 0$ and $\beta \in (0, 1)$. The current endowment is given by y_0.

In period 1 and 2, the household has a stochastic endowment $\underline{y}$ or $\bar{y}$. Uncertainty is described by an iid process that puts equal probability on the states.

(a) Find all AD prices in $t = 0$ for a unit of consumption in period 1 and 2 and in $t = 1$ for a unit of consumption in period 2.

(b) Find the price in $t = 0$ and $t = 1$ of risk-free pure discount bonds that mature in period 1 and 2.

(c) Find the price of a lemon tree in $t = 0$ and $t = 1$ that pays 2 units of consumption in the high state and 1 unit of consumption in the low state.

(d) Find the price of an orange tree at $t = 0$ and $t = 1$ that pays 1 unit of consumption in the high state and 2 units of consumption in the low state.

(e) Compare the prices of the four assets in period 0. Explain their differences.

(f) Assume now that there is a single tree owned by the household that pays out the endowment $y_t \in \{1, 2\}$ as a dividend to its owner in periods $t = 0, 1, 2$. There is no other endowment. Find the unconditional equity premium at $t = 0$.

[Hint: The equity premium is defined as the expected return of the tree minus the one period risk-free rate.]

[Hint: The unconditional equity premium averages the premium across possible states in period 0.]

(g) Find the price of a call option on the tree in period 0 where the strike price is equal to the price of the tree in state $\underline{y}$ in period 1. [Hint: This question is optional.]

4. Consider the following two period economy. There is a household with preferences over consumption and leisure that are again given by

$$u(c, 1-n) = \frac{c^{1-\gamma}}{1-\gamma} + \theta \frac{(1-n)^{1-\eta}}{1-\eta}$$

in each period with the household's endowment of time being normalized to 1. For simplicity assume that there is no discounting ($\beta = 1$). The household has an endowment of time equal to 1 in each of the two periods so that $n \in [0, 1]$.

There is a firm that carries out production according to

$$y_t = n_t^\alpha.$$

in each period.

There is a government that needs a total of g resources to build useless pyramids at the end of the two periods. The only way for the government to obtain resources is to tax people's labour income. It can levy taxes (τ_1, τ_2) to obtain revenue, but cannot borrow or lend from or to the household. The government needs to ensure that it raises sufficient taxes to cover g over time. Hence, $g = g_1 + g_2$ and the resources it obtains in each period are not longer available for consumption, so that $c_t = y_t - g_t$.

(a) Formulate the government's budget constraint *in each period*, distinguishing between g_1 and g_2.

(b) Define a competitive equilibrium for this economy for any policy (τ_1, τ_2).

(c) Find the first-order conditions for the household's problem and the firm's problem as a function of taxes.

Assume now $\gamma = \eta = 1$ and set $\alpha = 1$.

(d) Find the competitive equilibrium for this economy given expenditures (g_1, g_2). What are the associated tax rates (τ_1, τ_2)?

(e) Given any total expenditure g, which policy (τ_1, τ_2) maximizes the utility of the household? Interpret your answer.

Computational Part:

Set now $\gamma = \eta = 2$, $\alpha = 0.5$ and $\theta = 1$. Fix $g_1 = g_2$. [Hint: This implies that equilibrium in the two periods will be identical.]

(f) Compute the equilibrium labour supply n, wage rate w, consumption c and utility u as a function of government expenditure τ. Plot your results. [Hint: Use the FOC to express labour supply as a function of the wage w and the tax τ. Then, use the equilibrium wage rate and the government budget constraint to solve for n. Note that there will be limits on what values g can take.]

(g) Find the revenue maximizing value of the tax τ and the associated revenue g for the government.