Economics 815 **Winter 2020**

Macroeconomic Theory **Thorsten Koeppl**

Assignment 1

(Due: Tuesday, January 21 – in class)

1. Find a raw data series for Canadian GDP.

   (a) Detrend the data by using

      (i) log-differences

      (ii) an HP filter with $\lambda = 1,600$

      (iii) an HP filter with $\lambda = 400$.

   (b) Based on a spectral density estimate plot a periodogram for

      (i) the original data series

      (ii) the log-differenced data series

      (iii) the HP filtered data series where $\lambda = 1,600$

      (iv) the HP filtered data series where $\lambda = 400$.

   (c) Which frequencies of fluctuations matter the most for these periodograms? Interpret your findings.

2. Consider the following two-period economy. There are two representative households which have preferences given by

$$u(c_1, c_2) = \ln c_1 + \beta \ln c_2.$$

   Household 1 has only and of endowment $y = 1$ in period 1 and household 2 has only an endowment $y = 1$ in period 2. These households can save or borrow across periods at interest rate $1 + r$.

(a) Find the competitive equilibrium for this economy and interpret your result. [Hint: Derive the intertemporal budget constraints from the sequential ones and use it to find the equilibrium.]

(b) What happens to interest rates and the equilibrium allocation when (i) $\beta = 1$, (ii) $\beta \to \infty$ and (iii) $\beta \to 0$? Interpret your results.

3. Time is discrete and given by $t = 1, 2, \ldots$ A person discounts the future with a discount factor of $\beta \in (0, 1)$ per period so that his/her utility is given by

$$E\left[\sum_{t=1}^{\infty} \beta^t u(c_t)\right]$$

where u is increasing and strictly convave and c_t is consumption received by the person in period t. Consider the following two lotteries for the person:

(i) obtain one unit of consumption for sure in every even period;

(ii) obtain one unit of consumption with probability 1/2 before every even period and with probability 1/2 after every even period.

Which lottery does the person prefer? Prove your answer.

4. Supppse interest rates are given by $\beta(1 + r) = 1$ and consider a consumer that has quadratic preferences so that his Euler equation is given by

$$E_t(c_{t+1}) = c_t.$$

The consumer's income y_t follows an AR(1) process

$$y_t = \rho y_{t-1} + \epsilon_t$$

where $\rho \in (0, 1)$ and $E_{t-1}\epsilon_t = 0$.

(a) Derive the consumption function in terms of assets and the expected net present value of income. [Hint: $E_t y_{t+j} = \rho^j y_t$. Why?]

(b) Express the consumption function as a function of last period's consumption and the net present value of future income. [Hint: Lag the consumption function by one period to find c_{t-1} and use the $t-1$ budget constraint to get rid of a_t.]

(c) How is the variance of the consumption growth rate related to the income risk as given by the variance of income shocks ϵ_t?