ECON 815

Calibration

Winter 2020

Parameter Values

How do we pick the parameters for our model?

1) Inspection and sensitivity analysis

- γ
- η

We start off either with the "log" case $\gamma = \eta = 1$ – or $\gamma = 1$ and the indivisible labour model.

2) Match steady state to moments in the data

- β
- α
- δ
- θ

3) Estimate shocks

- ρ
- σ

Matching Moments

$$1 = \beta \left(f_k + (1 - \delta) \right)$$

Discount Factor β:

- We match the long-run return of capital.
- For example, take the long-run average annual return R on a stock market index.
- Since we deal with quarterly data, we approximately have $\beta = \frac{1}{1+R/4}$.

We presume that stock market returns are gross returns (include depreciation).

$$\bar{c} + \bar{k} = \bar{z}F(\bar{k}, \bar{n}) + (1 - \delta)\bar{k}$$

Depreciation δ:

- between 5% and 10%
- quarterly we have $\delta = 0.025$

We normalize $\bar{z}$ to 1, since we are only interested in fluctuations around the steady state and not the level.

This allows us to rewrite

$$\frac{\bar{c}}{\bar{y}} = 1 - \delta\frac{\bar{k}}{\bar{y}}$$

Data on the consumption share allows us to pin down the capital-output ratio.

$$\left(\frac{\bar{c}^{-\gamma}}{\theta(1-\bar{n})^{-\eta}}\right) = \frac{1}{f_n}$$

Labour input:

- people spend about 20-30% of their (available) time working
- hence we have in steady state $\bar{n} \in [0.2, 0.3]$

Labour Share $1-\alpha$:

- from the income side of the national accounts
- roughly between 50% and 67% of total income
- α falls in between 1/3 and 1/2

Weight θ:

- use $\gamma = \eta = 1$ to calculate from the FOC for labour-leisure choice
- it is good practice to recalibrate θ when changing γ or η
- for $\gamma \neq \eta \neq 1$ this is not straightforward

Solow Residuals and Shocks

Consider the production function

$$Y_t = z_t F(K_t, N_t X_t)$$

where $X_t = \gamma X_{t-1}$ with $\gamma > 1$.

The *Solow residuals* are measured by

$$\log SR_t = \log z_t + (1-\alpha)\log X_t = \log Y_t - \alpha \log K_t - (1-\alpha)\log N_t.$$

We assume that $\log z_t$ is AR(1) with mean 0 and that $\log X_t$ has a deterministic trend.

- Solow residual inherits the trend.
- The productivity shock is just the deviations from this trend.

Estimating Productivity Shocks

Step 1:

Calculate Solow Residuals

$$\log SR_t = \log Y_t - \alpha \log K_t - (1-\alpha) \log N_t$$

Step 2:

Fit a linear trend to the Solow Residual. This captures productivity growth $\gamma^t X_0$.

Step 3:

Take out the residuals from fitting the linear trend and use them to estimate ρ and σ.

Detrending Labor Productivity

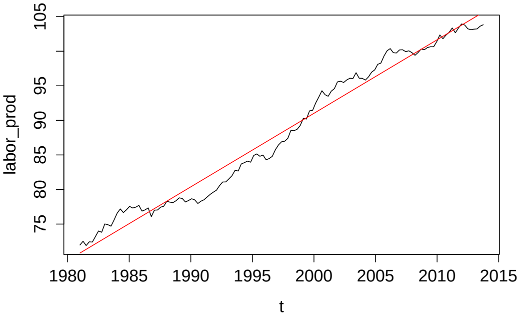

Figure: Labor Productivity – 1981:1 - 2013:3

Issue: no data off the shelf for multifactor (incl. capital) productivity

Next, we fit the residuals from the detrended series to an AR(1) process (with intercept).

We obtain

- $\rho = 0.9562$
- $\sigma = 0.004824$

For US data, people usually assume that

- $\rho_{US} \in [0.95, 0.98]$
- $\sigma_{US} \in [0.005, 0.01]$

A “heroic” assumption is that the properties of TFP shocks are constant across different economies (but not TFP levels).

Obtaining Productivity Shocks Directly

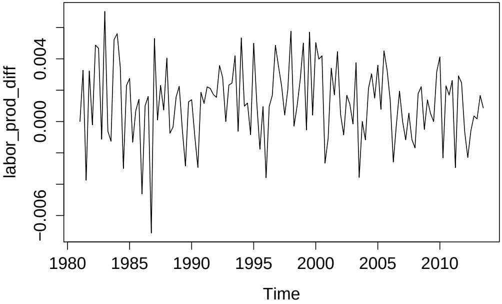

Figure: Log-differences Labor Productivity – 1981:1 - 2013:3

This yields as an estimate $\sigma = 0.00246$.