ECON 815

The Canonical RBC Model

Winter 2020

Model

- infinite horizon: $t = 0, 1, 2, \ldots$
- preferences

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U(c_t, 1 - n_t) \right]$$

- endowments
  - initial level of capital k_0
  - one unit of time each period
- production
  - firms have a neoclassical production function as before: $z_t F(k_t, n_t)$
  - capital depreciates at rate δ
  - Technology shocks: $\ln z_t = (1 - \rho) \ln \bar{z} + \rho \ln z_{t-1} + \epsilon_t$ with $\rho \in (0, 1)$ and $\epsilon_t \sim \mathcal{N}(0, \sigma^2)$

Note: z_t is the only exogenous variable in the economy.

Technology Shock

We have that $\ln z_{t+1}$ is normally distributed with

- mean $E_t = \mu_t = (1-\rho)\ln \bar{z} + \rho \ln z_t$
- variance $Var_t = \sigma^2$

The long-run, unconditional distribution of $\ln z$ is given by a normal distribution with

- mean $E[\ln z] = \mu = \ln \bar{z}$
- variance $Var[\ln z] = \frac{\sigma^2}{1-\rho^2}$

This implies that productivity z is **log-normally** distributed with

- mean $E[z] = e^{\mu + \frac{1}{2}\sigma^2}$
- variance $Var[z] = (e^{\sigma^2} - 1)E[z]^2$

We can normalize $\bar{z} = 1$ so that $\ln \bar{z} = 0$.

Parameters

Preferences:

$$\frac{c^{1-\gamma}}{1-\gamma} + \theta \frac{(1-n)^{1-\eta}}{1-\eta}$$

- elasticities of substitution – $1/\eta$ and $1/\gamma$
- weight θ
- discount factor β

Technology:

- α
- δ
- $\bar{z}$

Shocks (need to be estimated):

- serial autocorrelation ρ
- variance of shock σ

Firm's Problem

$$\max_{k,n} z_t F(k_t, n_t) - w_t n_t - r_t k_t$$

FOC:

$$z_t \alpha \left(\frac{k_t}{n_t}\right)^{\alpha-1} = r_t$$
$$z_t (1-\alpha) \left(\frac{k_t}{n_t}\right)^{\alpha} = w_t$$

Zero profits, but **factor prices depend on the current state z_t**.

Household's Problem

$$\max_{c_t,k_t,n_t} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\gamma}}{1-\gamma} + \theta \frac{(1-n_t)^{1-\eta}}{1-\eta} \right) \right]$$

subject to

$$c_t + x_t \leq w_t n_t + r_t k_t \text{ for all } t \text{ and } z_t$$

$$k_{t+1} = x_t + (1-\delta) k_t$$

$$k_0 \text{ and } z_0 \text{ given}$$

FOC:

$$\left(\frac{c_t^{-\gamma}}{\theta(1-n_t)^{-\eta}} \right) = \frac{1}{w(z_t)} \text{ for all } t \text{ and } z_t$$

$$1 = E\left[\beta \left(\frac{c_t}{c_{t+1}} \right)^{\gamma} (r_{t+1} + (1-\delta)) \middle| z_t \right] \text{ for all } t \text{ and } z_t$$

$$c_t + k_{t+1} = w_t(z_t) n_t + r_t(z_t) k_t + (1-\delta) k_t \text{ for all } t \text{ and } z_t$$

Steady State

Suppose that $z_t = \bar{z}$ for all t.

From the firm's problem and market clearing, we obtain that the steady state is described by

$$\left(\frac{\bar{c}^{-\gamma}}{\theta(1-\bar{n})^{-\eta}}\right) = \frac{1}{f_n}$$
$$1 = \beta\left(f_k + (1-\delta)\right)$$
$$\bar{c} + \bar{k} = \bar{z}F(\bar{k}, \bar{n}) + (1-\delta)\bar{k}$$

We have three equations in three unknowns that we can solve.

The dynamics are way more tricky.

Why? The intertemporal Euler equations is a non-linear, second-order difference equation.

Consumption function

Using $1 = \beta(\bar{r} + (1-\delta))$, we obtain

$$1 = E_t\left[\left(\frac{c_t}{c_{t+1}}\right)^{\gamma}\left(\frac{r_{t+1} + (1-\delta)}{\bar{r} + (1-\delta)}\right)\right]$$

With certainty equivalence, we have approximately

$$E_t\left[\ln\left(\frac{c_{t+1}}{c_t}\right)\right] \approx \frac{1}{\gamma}\left(E_t[r_{t+1}] - \bar{r}\right).$$

Expected consumption growth depends on

- expected changes in return to capital
- willingness to substitute intertemporally
- ... and higher moments of uncertainty (which we have neglected).

As $E_t[r_{t+1}]$ increases, it must be the case that the growth rate of consumption increases.

What about current consumption c_t?

1) There is a **substitution effect**, as the opportunity cost of consumption today increases.

2) There is a positive **income effect**, as the household becomes richer with higher interest rates.

The overall effect depends on the strength of these two effect.

For $\gamma = 1$, they exactly cancel out. For $0 < \gamma < 1$ ($\gamma > 1$) the substitution (income) effect dominates.

Labour Supply Decisions

Recall that

$$\left(\frac{c_t^{-\gamma}}{\theta(1-n_t)^{-\eta}}\right) = \frac{1}{w_t}.$$

Now people can change their labour supply in response to shocks which enter through w_t directly.

Usually, we think about an **extensive** and not an **intensive** margin.

- work is fixed at $h < 1$ hours
- fraction ψ of people work, others do not
- assume that people insure individual consumption risk

This changes utility to be *linear* in labour

$$\psi\left(\frac{c_t^{1-\gamma}}{1-\gamma} + \theta\frac{(1-h)^{1-\eta}}{1-\eta}\right) + (1-\psi)\left(\frac{c_t^{1-\gamma}}{1-\gamma} + \theta\frac{1^{1-\eta}}{1-\eta}\right) = \frac{c_t^{1-\gamma}}{1-\gamma} + \tilde{\psi}\frac{(1-h)^{1-\eta}-1}{1-\eta}$$

How Do We Proceed Now?

1) How do we pick the parameters for the model?

$\Longrightarrow$ Calibration and Estimation

2) How do we analyze the dynamics of the model?

$\Longrightarrow$ Linear first-order difference equation for the model

3) How do we judge how well the model does?

$\Longrightarrow$ Simulation and Impulse Response Functions

But what happened to trend growth?

We can detrend a model with growth and work with that model.

Or we start out from a model without trend as shown here.

Of course, this requires us to also work with detrended data.