ECON 815

Long-run Debt

Winter 2020

Two Main Questions

1) What level of public debt is sustainable?

fiscal costs & feasibility

2) What level of public debt is optimal?

welfare costs & efficiency

The first question is about the gov't intertemporal budget constraint.

The second question is about the special nature of gov't debt.

We are not talking about countercyclical fiscal policy here.

Gov't Intertemporal Budget Constaint

We call the primary deficit as a percentage of GDP $\Delta = \frac{G-T}{Y}$.

The law of motion for the Debt/GDP ratio is given by

$$B_{-1} + G = \frac{B_0}{1+r} + T$$

$$\frac{B_{-1}}{Y_{-1}(1+\gamma)} = \frac{B_0}{Y_0(1+r)} - \Delta(1+r)$$

$$b_0 = b_{-1}\left(\frac{1+r}{1+\gamma}\right) + \Delta(1+r)$$

The solution is given by

$$b_t = \left(\frac{1+r}{1+\gamma}\right)^{t+1}\left(b_{-1} - \Delta(1+r)\left(\frac{1+\gamma}{\gamma-r}\right)\right) + \Delta(1+r)\left(\frac{1+\gamma}{\gamma-r}\right)$$

Hence, what matters for the fiscal costs is again r vs. γ.

Some Unpleasant Arithmetic? $r \geq \gamma$

We have that $\left(\frac{1+r}{1+\gamma}\right)^{t+1} \to \infty$.

Hence, for debt to be sustainable, we need

$$\frac{b_{-1}}{1+r} = \Delta\left(\frac{1+\gamma}{\gamma - r}\right)$$

Implications:

$b_{-1} > 0$ requires $\Delta = (G - T)/Y < 0$.

Debt level is not sustainable, unless one runs primary surpluses.

Any increase in debt cannot be rolled over indefinitely.

If the spread $r - \gamma$ increases, one needs to run larger primary surpluses to keep debt sustainable.

Paradise for Politicians? $r < \gamma$

We have that $\left(\frac{1+r}{1+\gamma}\right)^{t+1} \to 0.$

Hence, in the long-run debt converges to

$$\frac{b_\infty}{1+r} = \Delta \left(\frac{1+\gamma}{\gamma - r}\right)$$

Implications:

Long-run debt level depends positive on the primary deficit.

Debt looks like an AR(1) process with coefficient $(1+r)/(1+\gamma)$ where changes in the primary deficit are like shocks.

Warning: This neglects endogenous responses to interest rates and growth rates! And uncertainty & risk aversion.

Canadian Data (1962-2018)

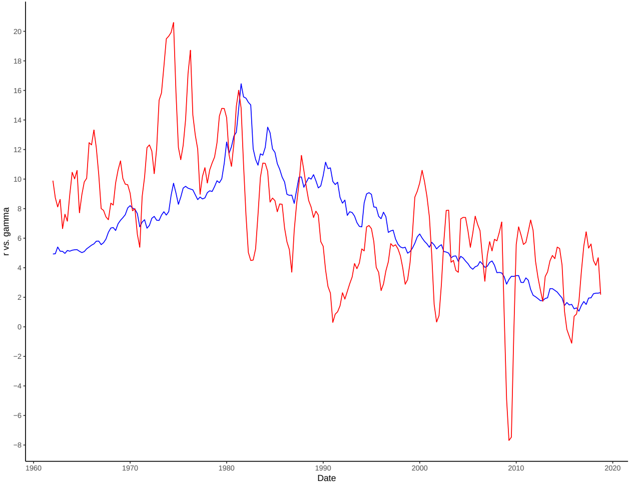

Figure: 10-Year Bond Yields (blue) vs. Nom. GDP Growth (red)

Debt w/ OLG

People live for two period, work & save in the first and only consume in the second period.

$$
\begin{aligned}
&u(c_1, c_2) = (1-\beta)\log c_1 + \beta \log c_2\\
&\text{subject to}\\
&\qquad c_1 = w - k - d\\
&\qquad c_2 = rk + d
\end{aligned}
$$

The variable d captures "debt", but is modelled as a transfer from the young to the old of *different generations*.

We assume no growth. Thus transfers between generations are feasible each period.

Important: Think of transfers and capital as two different technologies.

Indirect Utility and Transfers

The FOC (in equilibrium) is given by

$$\frac{1-\beta}{\beta} = r\frac{c_1}{c_2} = r\frac{w+k-d}{rk+d}$$

Changing transfers d will change equilibrium prices.

How? Production function matters!

We have

$$\begin{aligned}\frac{dU}{dd} &= -\frac{1-\beta}{c_1} + \frac{\beta}{c_2} + \frac{1-\beta}{c_1}\frac{dw}{dd} + \frac{\beta k}{c_2}\frac{dr}{dd} \\ &= (1-r)\frac{\beta}{c_2} + \frac{\beta}{c_2}\left(r\frac{dw}{dd} + k\frac{dr}{dd}\right)\end{aligned}$$

Assume that production is Cobb-Douglas with $N = 1$. Then,

$$dw = -kdr$$

Hence, the change in utility from a change in transfers is given by

$$dU = (1 - r)\frac{\beta}{c_2}dd + \frac{\beta}{c_2}(1 - r)kdr$$

Note that $dr/dd > 0$, since transfers and investments are substitutes for people to save.

<u>Result:</u> If $r < 1$, transfers are better than capital accumulation.

This results comes from a dynamic inefficiency where people save too much (self-insurance) rather than relying on the government providing a better meachanism (social insurance).

With debt, things are more complicated as interest rates need to equal to the MPK in equilibrium. Hence, the above is only a local argument.

Further Remarks

1) The optimal level of capital is achieved if $r = 1$.

2) Hence, a stationary level of debt such that $r = 1$ will yield the optimal level of capital.

3) With uncertainty, one can show that the first-term depends on the risk-free rate, but the second one on the risky MPK so that the terms can have opposite signs.

4) In a more general formulation, the second term depends inversely on the elasticity of substitution $\eta = 1/(1-\rho)$ between capital and labour.

$$F(K,N) = A\left(bK^{\rho} + (1-b)N^{\rho}\right)^{\frac{1}{\rho}}$$

A high elasticity implies a larger level of debt.