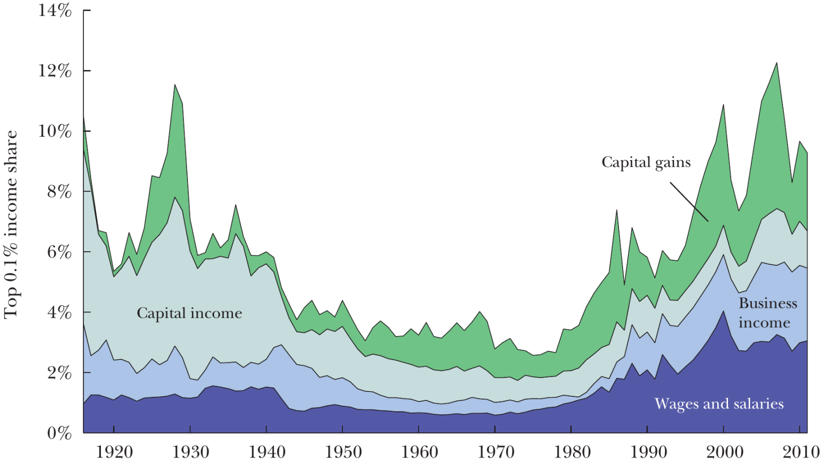
ECON 815

Long-run Inequality

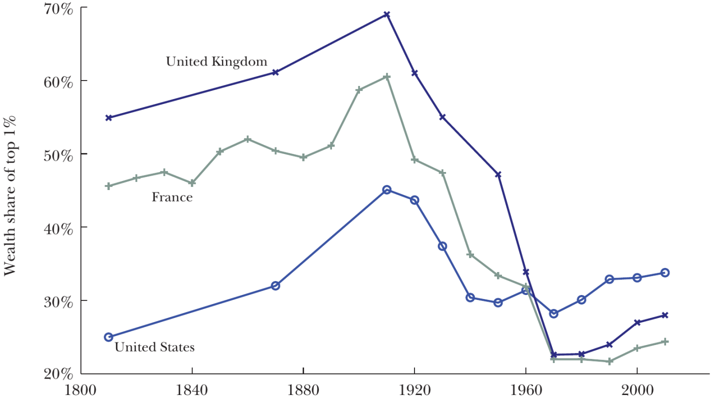
Winter 2020

Some Facts – Income Inequality



Source: These data are taken from the “data-Fig4B” tab of the September 2013 update of the spreadsheet appendix to Piketty and Saez (2003).

Some Facts – Wealth Inequality



Source: Supplementary Table S10.1 for chapter 10 of Piketty (2014), available at: http://piketty.pse.ens.fr/capital21c.

Note: The figure shows the share of aggregate wealth held by the richest 1 percent of the population.

Pareto's Principle

Let the number of people N_y with income above y be given by a **power law**

$$N_y = Ay^{-\frac{1}{\eta}}$$

with $0 < \eta \leq 1$.

The proportion of people earning income above y is then

$$\frac{N_y}{N_0} = \left(\frac{y}{y_0}\right)^{-\frac{1}{\eta}} = y^{-\frac{1}{\eta}}$$

where we have normalized the minimum income to $y_0 = 1$.

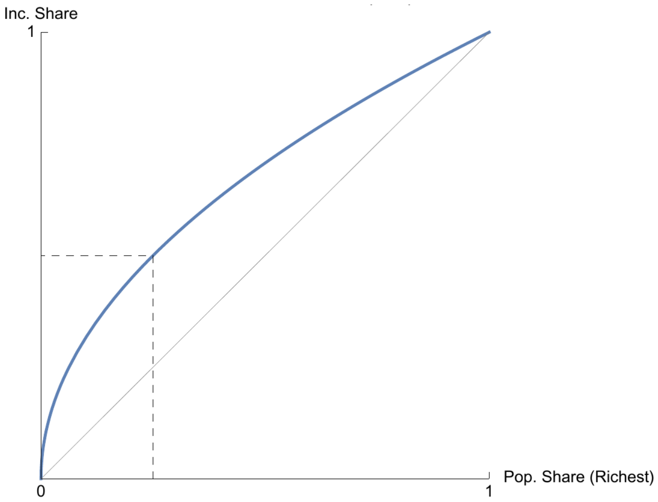
This is identical to the survival function of a **Pareto distribution**

$$F(x) = 1 - x^{-\alpha}$$

for $x \geq 1$.

We call η the **coefficient of Pareto inequality**.

Note that any η gives us a **Lorenz Curve** and a **Gini Coefficient** that measures inequality.



Why?

Total income above $\tilde{y}$ is given by

$$\int_{\tilde{y}}^{\infty} y dF(y) = \left(\frac{1}{1-\eta}\right) \tilde{y}^{(1-\frac{1}{\eta})}$$

where total income is given by $\left(\frac{1}{1-\eta}\right)$.

The share of income s going to the top $n\%$ is then given by

$$s = \tilde{y}^{(1-\frac{1}{\eta})} = n^{-\eta(1-\frac{1}{\eta})} = n^{1-\eta}$$

for $\eta \in (0, 1]$

For example, with $\eta = 0.6$, we get that the top 1% earn a share of 16% of income and the top 0.1% a share of 6.5%.

A Simple Model of Income Inequality

Trick! To get a Pareto distribution, we need to have *exponential growth* over an *exponentially distributed amount of time*.

Assumption 1: Talent is exponentially distributed so that

$$\mathcal{P}[Talent > x] = e^{-\delta x}$$

Assumption 2: Income increases exponentially with talent

$$y = e^{\mu x}$$

Income is then distributed according to

$$\mathcal{P}[Income > y] = \mathcal{P}[Talent > x(y)] = e^{-\delta x(y)} = y^{-\frac{\delta}{\mu}}$$

This is a Pareto distribution with $\eta = \frac{\mu}{\delta}$.

What could drive income inequality?

Any channel has to work through δ and/or μ.

1) The distribution of talent produces superstars that can take unique advantages of technology.

2) Some entrepreneurs have all the luck.

3) Taxes have become less progressive.

4) Inheritance and human capital (schooling).

Any policy responses will have to take into account the channel responsible for the upward trend in inequality.

A Simple Model of Wealth Inequality

Assumptions:

- wealth earns interest rate r and is taxed at rate τ
- people are born at rate b and die over time at rate d for growth n
- per capita assets grows at rate g

If consumption is a constant fraction of wealth, we have from the budget constraint that

$$\begin{aligned} a_{t+1} &= (1 + r - \tau)a_t - c_t \\ &= (1 + r - \tau - \alpha)a_t \end{aligned}$$

Hence,

$$a_t \simeq a_0 e^{(r-\tau-\alpha)t}$$

At **time t**, we have different **cohorts with age x** that – depending on when they were born – have different wealth levels.

Cross-section of Wealth

Person in period t, at age x, has **initial** (inherited) wealth given by

$$a_{t-x} = \frac{A_t}{N_t} e^{-gx}$$

Current wealth as a function of age x is given by

$$a_t(x) = \frac{A_t}{N_t} e^{-gx} e^{(r-\tau-\alpha)x}$$

The **cross section** of wealth is thus given by

$$x(a) = \left(\frac{1}{r-g-\tau-\alpha}\right) \log\left(\frac{a}{\bar{a}_t}\right)$$

What drives wealth inequality?

Wealth grows exponentially and age is exponentially distributed

$$\mathcal{P}[Age > x] = e^{-(n+d)x}$$

We have

$$\mathcal{P}[Wealth > a] = \mathcal{P}[Age > x(a)] = e^{-(n+d)x(a)} = \left(\frac{a}{\bar{a}k_t}\right)^{-\frac{n+d}{r-g-\tau-\alpha}}$$

This is a Pareto distribution with coefficient

$$\eta = \frac{r-g-\tau-\alpha}{n+d}$$

<u>So what?</u>

1) Piketty: $r >> g$ and increasing over time.

2) Slow down in population growth.

3) Lower top marginal tax rates.

General Equilibrium I

Assumption: Log-utility, taxes are used to build pyramids, $Y_t = AK_t$.

Since consumers have log utility, we get

$$(1+g) = \frac{c_{t+1}}{c_t} = (1+r-\tau)\beta$$

where consumption grows at rate g.

Hence,

$$(r-g-\tau) = (\theta + d)$$

where θ is the rate of time preference.

This implies that changes in taxes or growth move r one-for-one.

Conclusion: For wealth taxes to matter, we need a big substitution effect to reduce wealth inequality.

General Equilibrium II

We have that the aggregate economy evolves according to

$$Y_t = C_t + I_t + T_t = \left(\alpha + \frac{\dot{K}_t}{K_t} + \tau\right) K_t$$

The per capita growth rate is then given by

$$g = \left(\frac{\dot{Y_t}}{N_t}\right) / \frac{Y_t}{N_t} = A - \alpha - \tau - n$$

Using the fact that $r = A$, we get

$$\eta = \frac{n}{n+d}$$

Conclusion: This implies that taxes don't matter at all.

- incentive effects?
- elasticity of substitution in production?
- heterogenous preferences over kids?