ECON 815

Lucas Critique and Ricardian Equivalence

Winter 2020

Production with Labour and Capital

- firm hires labour n and pays wages w
- firm buys capital k at interest rate r
- Production function:

$$F(k,n) = Ak^{\alpha}n^{1-\alpha}$$

  with $A > 0$ and $\alpha \in (0,1)$.

This is a neoclassical production function.

- homogeneous of degree one – or CRS

$$F(\lambda k, \lambda n) = \lambda F(k,n) \text{ for all } \lambda > 0$$

- diminishing marginal products for all inputs
- Inada conditions
- for fixed labor input, decreasing returns to scale in capital

Optimal Production

Maximize Profits:

$$\max_{k,n} AF(k,n) - wn - rk$$

<u>Solution:</u>

$$\begin{aligned} \text{MPK} = F_k = A\alpha\left(\frac{k}{n}\right)^{\alpha-1} &= r \\ \text{MPL} = F_l = A(1-\alpha)\left(\frac{k}{n}\right)^{\alpha} &= w \end{aligned}$$

This implies zero profits.

The firm's output $AF(k,n)$ is just split between the inputs, labour and capital, according to the factor shares α and $(1-\alpha)$.

Labour vs. Leisure Choice

Preferences are defined over two goods, consumption and leisure

$$U(c_1, 1-n_1) + \beta U(c_2, 1-n_2) = u(c_1) + v(1-n_1) + \beta(u(c_2) + v(1-n_2))$$

Endowments:

- time $n_1 \in (0, 1)$
- time $n_2 \in (0, 1)$
- capital k_1 (and k_2), fully depreciates after production

Budget constraints – sequential:

$$\begin{aligned} c_1 + k_2 &\leq w_1 n_1 + r_1 k_1 \\ c_2 &\leq w_2 n_2 + r_2 k_2 \end{aligned}$$

Intertemporal budget constraint:

$$c_1 + \frac{c_2}{r_2} \leq w_1 n_1 + \frac{w_2 n_2}{r_2} + r_1 k_1$$

Solution

FOC:

$$\begin{aligned} \frac{u'(c_t)}{v'(1-n_t)} &= \frac{1}{w_t} \text{ for } t = 1, 2 \\ \frac{u'(c_1)}{\beta u'(c_2)} &= r_2 \end{aligned}$$

In terms of labour choice only, we get

$$\frac{v'(1-n_1)}{\beta v'(1-n_2)} = \left(\frac{w_1}{w_2}\right) r_2$$

How much people work depends on

- productivity today (w_1)
- (expected) productivity tomorrow (w_2)
- willingness to substitute leisure across time (βr_2)

Taxes and Distortions

- labour income taxes: τ_1^n and τ_2^n
- capital income taxes: τ_1^k and τ_2^k
- lump sum taxes: T_1 and T_2

The budget constraints then become

$$\begin{aligned} c_1 + k_2 &\leq (1-\tau_1^n) w_1 n_1 + (1-\tau_1^k) r_1 k_1 + T_1 \\ c_2 &\leq (1-\tau_2^n) w_2 n_2 + (1-\tau_2^k) r_2 k_2 + T_2 \end{aligned}$$

Wedges in FOCs:

$$\begin{aligned} \frac{u'(c_t)}{v'(1-n_t)} &= \frac{1}{(1-\tau_t^n) w_t} \text{ for } t = 1, 2 \\ \frac{u'(c_1)}{\beta u'(c_2)} &= (1-\tau_2^k) r_2 \end{aligned}$$

Lump sum taxes (T_1, T_2, τ_1^k) do not distort decisions.

Analyzing Changes in Taxes ...

Some assumptions to make our life easier.

- production function is linear in labour $f(n) = n$
- there is no investment, but people can save at interest rate $1 + r$

The intertemporal budget constraint is then given by

$$c_1 + \frac{c_2}{1+r} \leq (1-\tau_1)w_1 n_1 + \frac{(1-\tau_2)w_2 n_2}{1+r}$$

Taxes are used to build pyramids G which do not provide any utility.

- no lump-sum taxes
- tax revenue is given by $R_1 = \tau_1 w_1 n_1$ and $R_2 = \tau_2 w_2 n_2$
- government budget constraint $g_t = R_t = \tau_t w_t n_t$
- $G = g_1 + g_2$

... is not so Straightforward!

Key Idea: People's decisions are not fixed.

They react to changes in policy and anticipate future changes in policy.

In turn, when decisions change, equilibrium will also change.

The total change in current revenue dR_1 is given by

$$dR_1 = \left(w_1 n_1 + \tau_1 w_1 \frac{\partial n_1(\tau_1, \tau_2^e)}{\partial \tau_1} + \tau_1 n_1 \frac{\partial w_1(\tau_1, \tau_2^e)}{\partial \tau_1} \right) d\tau_1$$

- the first term is the effect of *changes in the tax rate*
- the second and third term are *changes in the tax base*
- these depend on how people's labour supply reacts to tax changes
- the variable $\tau_2^e(\tau_1)$ stands for *expected future tax changes*

The Lucas Critique

1) Decision rules are not invariant to policy changes.

$\Longrightarrow$ We need to use the first-order condition $\frac{u'(c_1)}{v'(1-n_1)} = \frac{1}{(1-\tau_1)w_1}$.

2) People can forecast the effects of policy changes and will adjust their behaviour appropriately.

$\Longrightarrow$ To finance G, it must be the case that τ_2 changes when τ_1 does.

3) There are feedback effects from individual decisions (general equilibrium effects).

$\Longrightarrow$ The equilibrium wage rate and output will change as people adjust their labour supply.

Conclusion: We need a structural model in order to be able to evaluate economic policy. The model has some structural parameters which are fixed, but people's decisions vary with economic policy.

Ricardian Equivalence

Some assumptions to make our life even easier.

- Labour is inelastically supplied, that is $n_1 = n_2 = 1$.
- Labour is still transformed 1-1 into output $f(n) = n$.

We require that the government needs to consume exactly g_1 and g_2.

It can raise labour taxes τ_t in both periods and borrow (or lend) b at rate $(1 + r)$ in the first period.

It is important here that borrowing/lending is from/to people.

Government budget constraints

$$\begin{aligned} g_1 &= \tau_1 w_1 n_1 + b \\ g_2 + (1+r)b &= \tau_2 w_2 n_2 \end{aligned}$$

Theorem: Let $(c_1, c_2, (1+r), w_1, w_2)$ be an equilibrium for government policies $(g_1, g_2, \tau_1, \tau_2, b)$. Then the same allocation and prices are still an equilibrium for any policy $(g_1, g_2, \tilde{\tau}_1, \tilde{\tau}_2, \tilde{b})$ that satisfies the government's budget constraints.

In other words, the government's timing of taxation (its debt policy) is irrelevant as long as spending remains the same.

Note that here taxes are lump-sum, i.e. they do not distort people's decisions.

If they were not, for changes in tax policy to be Ricardian equivalent one would need to leave distortions unchanged.

Proof:

In equilibrium, we have

$$\begin{aligned} \frac{u'(c_1)}{\beta u'(c_2)} &= (1+r) \\ w_1 &= w_2 = 1 \end{aligned}$$

Taking into account the government's budget constraints, the NPV budget constraint is given by

$$\begin{aligned} c_1 + \frac{c_2}{1+r} &= (1-\tau_1)w_1 n_1 + \frac{(1-\tau_2)w_2 n_2}{1+r} \\ &= w_1 n_1 - g_1 + b + \frac{w_2 n_2 - g_2 - b(1+r)}{1+r} \\ &= w_1 - g_1 + \frac{w_2 - g_2}{1+r} \end{aligned}$$

The same interest rate and the same allocation still solve the consumer's problem and are, thus, an equilibrium.