ECON 815

Optimal Monetary Policy

Winter 2020

Long-run Wedge in Steady State

Efficient allocation:

$$\frac{C^\sigma}{(1-N)^\eta} = \frac{W}{P} = \alpha A N^{\alpha-1}$$

Monopolistic competition:

$$\frac{C^\sigma}{(1-N)^\eta} = \frac{W}{P} = \frac{\epsilon-1}{\epsilon}\alpha A N^{\alpha-1}$$

so that labour is paid less than its marginal product.

There is a wedge in the labour supply condition similar to a tax on labour income.

A subsidy for labour (financed by a lump-sum tax) can remove this inefficiency and achieve the efficient level of output. It is given by

$$\frac{\epsilon-1}{\epsilon}(1+\tau) = 1.$$

Short-run Wedge

Shocks lead to (aggregate) variation in the desired average mark-up.

$$\mu_t = \frac{P_t}{W_t} \alpha A_t N_t^{\alpha-1} \neq \mu$$

Sticky prices lead to differences in demand across goods.

$$C_t(i) \neq C_t(j)$$

Key Idea: We need two policy instruments.

1) A time-invariant tax removes the long-run distortion.

2) Monetary policy removes short-run distortions due to sticky prices.

The Divine Coincidence

Assumption:

There is $\tau > 0$ so that $y_t^n = y_t^*$ for all t if prices were flexible.

Monetary policy only needs to aim at stabilizing the average mark-up μ_t perfectly across firms.

Why? Firms have no reason to change prices. Sticky prices do not matter anymore.

How does MP do this?

1. Choose nominal interest rate so that $i_t = r_t^n = r_t^*$.
2. This implies that the output gap is zero in equilibrium forever.
3. Phillips curve implies zero inflation.

Problem I – Multiple Solutions

For $i_t = r_t^n$, we have

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \begin{pmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{pmatrix} \begin{bmatrix} E_t[x_{t+1}] \\ E_t[\pi_{t+1}] \end{bmatrix}$$

Unique (stationary) solution requires that the EV of the matrix are both less than 1.

But only one is, so that there are multiple (stable) solutions.

Sunspots are possible, implying that monetary policy cannot determine inflation for sure.

A Taylor Rule Helps!

Consider now the Tayor rule

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_x x_t + v_t$$

where

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0.$$

Then the equilibrium is determinate.

<u>Intuition:</u>

The real interest will rise when inflation goes up. Neglecting dynamics and using the NK Phillips curve we have

$$\begin{aligned} di &= \phi_\pi d\pi + \phi_y dx \\ &= \phi_\pi d\pi + \left(\frac{\phi_y(1-\beta)}{\kappa}\right) d\pi > d\pi \end{aligned}$$

Problem II – r^n is not observable

Suppose a central bank can only observe inflation.

Consider welfare losses given by

$$W = \frac{1}{2} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{\kappa}{\lambda} x_t^2 + \frac{\epsilon}{\lambda} \pi_t^2 \right) \right]$$

or in form of an **average per-period loss function**

$$L = \frac{1}{2} \left[\frac{\kappa}{\lambda} Var[x_t] + \frac{\epsilon}{\lambda} Var[\pi_t] \right]$$

where $\kappa/\lambda = \sigma + \frac{\eta + (1-\alpha)}{\alpha}$ and losses are to be understood as % deviations from steady state.

Note: Sticky prices – i.e., parameters ϵ and θ – only influence the inflation variability term!

Strict Inflation Targeting Helps!

Consider now the rule

$$\begin{aligned} i_t &= \rho + \phi_\pi \pi_t + \phi_y (\log Y_t - \log Y_{SS}) \\ &= \rho + \phi_\pi \pi_t + \phi_y (\log Y_t - \log Y_t^n + \log Y_t^n - \log Y_{SS}) \\ &= \rho + \phi_\pi \pi_t + \phi_y x_t + \phi_y v_t \end{aligned}$$

where Y_{SS} is steady state output level.

Welfare losses are decreasing in ϕ_π, but increasing in ϕ_y.

By reacting sufficiently strongly to inflation, this rule can reduce fluctuations in the unique (!) equilibrium, and, hence, is optimal. To the contrary, reacting to deviations in output from steady state just add extra fluctuations in this model.

$\Longrightarrow$ **Strict Inflation Targeting**

One does neither need to measure an output gap nor react to it.

A Trade-off for Monetary Policy Problem

Suppose now that monetary policy needs to also address other deviations from the flexible price equilibrium.

Define $x_t = y_t - y_t^*$ as the output gap.

One can show that welfare losses are given by

$$W = \frac{1}{2} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \frac{\kappa}{\epsilon} x_t^2 \right) \right]$$

Rewriting the Phillips Curve relationship, we obtain

$$\begin{aligned} \pi_t &= \beta E_t[\pi_{t+1}] + \kappa(y_t - y_t^n + y_t^* - y_t^*) \\ &= \beta E_t[\pi_{t+1}] + \kappa(y_t - y_t^*) + u_t \end{aligned}$$

The variable $u_t = \rho_u u_{t-1} + \epsilon_t$ is called a **cost-push shock**. No clue what this is!

There is now an inflation-output trade-off for the central bank when setting interest rates.

The central bank solves

$$\min_{x_t,\pi_t} W$$

$$\text{subject to}$$

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa(y_t - y_t^*) + u_t$$

$$u_t = \rho_u u_{t-1} + \epsilon_t$$

To implement the optimal inflation and output gap, one sets i_t to satisfy

$$E_t[x_{t+1}] - x_t = \frac{1}{\sigma}\left(i_t - E_t[\pi_{t+1}] - (\rho + \sigma E_t[y_{t+1}^* - y_t^*])\right)$$

where the last term is often called the **efficient interest rate**.

APPENDIX

Digression – Blanchard & Kahn (1980)

Stable rational expectations equilibria

$$E_t[y_{t+1}] = Ay_t + B\epsilon_{t+1}$$

- y is a vector of state (backward looking) and control (forward looking) variables
- ϵ is shocks

Need to look at eigenvalues of A that are greater than 1.

Case 1 – Uniqueness and Saddle-Path Stability: number of EV greater than 1 equal to number of control variables

Case 2 – Multiplicity: not enough EV larger than 1

Case 3 – No stable solution: too many EV larger than 1