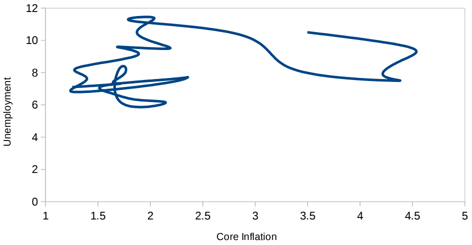
ECON 815

The Basic New Keynesian Model II
The Phillips Curve

Winter 2020

Unemployment vs. Inflation



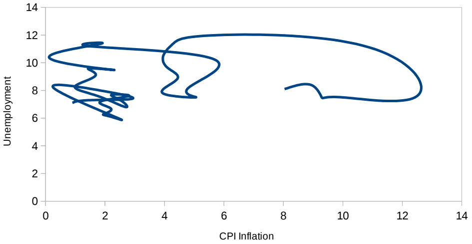
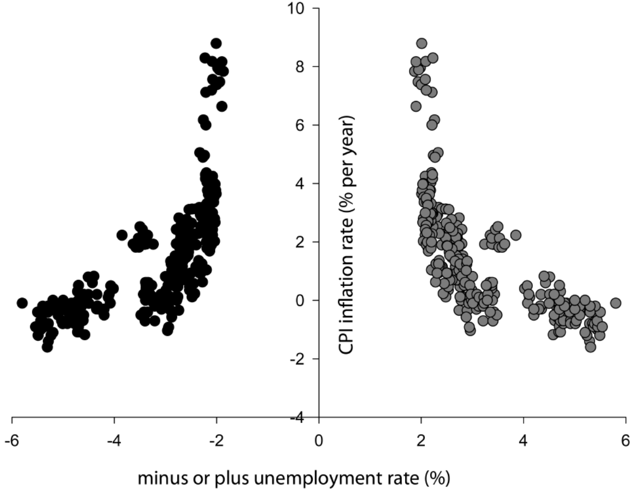


Figure: Unemployment vs. Inflation – Canada 1977 - 2013

Japan


10
8
6
4
2
0
-2
-4
CPI inflation rate (% per year)
-6
-4
-2
0
2
4
6
minus or plus unemployment rate (%)

Steady State

Suppose we have zero inflation in steady state, so that there is no need to change prices:

$$\begin{aligned} Y(i) &= Y \\ Y &= AN^\alpha \\ \frac{U_l}{U_c} &= \frac{W}{P} \\ 1 &= \beta(1+i) \\ \alpha\frac{Y}{N}\frac{P}{W} &= \frac{\epsilon}{\epsilon-1} \end{aligned}$$

This is the RBC model with constant mark-ups at

$$\mu = \frac{\epsilon}{\epsilon - 1}$$

that are equal to the inverse of real marginal costs

$$\frac{\varphi(i)}{P} = \frac{WN}{\alpha Y}\frac{1}{P}.$$

Sticky Prices

Firms can change their price only with probability $(1-\theta)$.

- Suppose they set their price in period t.
- In period $t+1$, with probability θ they cannot change it.
- In period $t+2$, conditional on not having changed in period 1, they cannot change it with probability θ.
- And so on ...

Firms solve:

$$\max_{P_t(i)} \sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} \left(P_t(i) Y_{t+k}(i) - W_{t+k} \left(\frac{Y_{t+k}(i)}{A_{t+k}} \right)^{\frac{1}{\alpha}} \right) \right]$$

subject to

$$Y_{t+k}(i) = \left(\frac{P_t(i)}{P_{t+k}} \right)^{-\epsilon} C_{t+k}$$

where $Q_{t,t+k}$ captures "stochastic equilibrium discounting".

FOC:

$$\sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} Y_{t+k}(i) \left(P_t(i) - \frac{\epsilon}{\epsilon - 1} \varphi_{t+k}(i) \right) \right] = 0$$

All firms that can change prices today, will chose the same price, P_t^* that gives on average a mark-up of μ.

Hence, firms take into account expected changes in nominal marginal costs when setting their price.

If the price cannot be adjusted throughout period $t + k$, the firm however is stuck at P_t^* and its actual mark-up is given by

$$\mu_{t+k} = \frac{P_t^*}{\varphi_{t+k}(i)} \neq \mu = \frac{\epsilon}{\epsilon - 1}.$$

Distortion

When the price level P_{t+k} increases (decreases), the demand for a firm with sticky price P_t^* increases (decreases) as its product is relatively cheap (expensive).

Hence, mark-ups are depressed (expand) as the nominal marginal costs increase (decrease) with production.

This can be interpreted as a time-varying labour wedge as mark-ups move in the opposite direction with labour demand.

Hence, unexpected price increases (decreases) will increase (decrease) output in the short-run.

$\Longrightarrow$ **New Keynesian Phillips Curve**.

Inflation Dynamics

We look at approximations around a zero inflation steady state.

For inflation, we obtain from the firm's FOC

$$\begin{aligned} \pi_t &= \beta E_t[\pi_{t+1}] + \lambda \left(\log \frac{\bar{\varphi}_t}{P_t} - \log \frac{\epsilon - 1}{\epsilon} \right) \\ &= \lambda \sum_{t=0}^{\infty} \beta^k E_t \left[\log \frac{\bar{\varphi}_{t+k}}{P_{t+k}} - \log \frac{\epsilon - 1}{\epsilon} \right] \end{aligned}$$

where

- $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{\alpha}{\alpha+\epsilon(1-\alpha)}$
- $\frac{\bar{\varphi}_t}{P_t}$ are average real marginal costs for firms

Inflation is given by *expected* deviations from steady-state mark-up.

Inflation is high (low) whenever firms expect real marginal costs above (below) their steady state values.

The New Keynesian Philips Curve

We can express deviations in real marginal costs in terms of deviations in output.

$$\log \frac{\bar{\varphi}_t}{P_t} - \log \frac{\epsilon - 1}{\epsilon} = \left(\sigma + \frac{\eta + (1-\alpha)}{\alpha} \right) (y_t - y_t^n)$$

The last expression is the *output gap* which measures the deviation of the actual output level from the output level associated with flexible prices, y_t^n, which is not (!) the optimal output level.

This links inflation dynamics to dynamics in output, or

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa(y_t - y_t^n)$$

where $\kappa = \lambda \left(\sigma + \frac{\eta + (1-\alpha)}{\alpha} \right)$.

Key Parameters

The parameter κ expresses the *slope* of the Phillips curve.

- If $\theta \to 1$, prices are very sticky.

  Hence, $\lambda(\kappa) \to 0$ and output gaps cause little inflation.

  $\implies$ "steep" Phillips curve

- If $\theta \to 0$, prices are very flexible.

  Hence, $\lambda(\kappa) \to \infty$ and output gaps cause a lot of inflation.

  $\implies$ "flat" Phillips curve

What is the role of ϵ?

As $\epsilon \to \infty$, we have that mark-ups disappear (perfect competition), as goods are more substitutable. Hence, firms move prices by less to attract demand (larger strategic complementarities).

Summary – The NK Trinity

1) Phillips Curve

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa(y_t - y_t^n)$$

2) Intertemporal Euler Equation

$$y_t - y_t^n = E_t[y_{t+1} - y_{t+1}^n] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - r_t^n)$$

where we have used $c_t = y_t$ and $r_t^n = \rho + \sigma E_t[y_{t+1}^n - y_t^n]$ is the natural rate of interest which changes due to real (or supply) shocks.

3) Monetary Policy Rule for i_t

The Phillips Curve specifies inflation in terms of the output gap which is given by the Euler equation through the natural rate and the actual real rate. The latter one is pinned down by monetary policy.

APPENDIX

Movements in Mark-ups

A firm that is stuck at price P_t^* sets labour demand to satisfy its demand for goods

$$N_{t+k}(i) = \left(\frac{Y_{t+k}(i)}{A_{t+k}}\right)^{\frac{1}{\alpha}} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\frac{\epsilon}{\alpha}} \left(\frac{C_{t+k}}{A_{t+k}}\right)^{\frac{1}{\alpha}}$$

For fixed P_t^* we have $\partial N_{t+k}(i)/\partial P_{t+k} > 0$, so that nominal marginal costs

$$\varphi_{t+k}(i) = \frac{W_t N_{t+k}(i)}{\alpha Y_{t+k}(i)}$$

are large.

In other words, mark-ups are depressed, i.e. $\mu_t < \mu$ or, equivalently, real marginal costs are high.

Aggregate Price Level

The price index in period t is given by

$$P_t^{1-\epsilon} = \int_{i|fixed} P_{t-1}(i)^{1-\epsilon} di + (1-\theta) P_t^{*1-\epsilon}$$

The distribution of fixed prices corresponds to the distribution of last periods prices with weight θ, or

$$\theta P_{t-1}^{1-\epsilon} = \int_{i|fixed} P_{t-1}(i)^{1-\epsilon} di$$

Hence, inflation is given by

$$\Pi_t = \left[\theta + (1-\theta) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$$

Inflation changes less than 1-1 with price changes of individual firms.