ECON 815

The Basic New Keynesian Model I
Aggregate Demand

Winter 2020

Overview

We will make two changes to the classical monetary model.

1. Monopolistic competition $\Longrightarrow$ demand determined equilibrium

2. Sticky prices $\Longrightarrow$ some firms cannot adjust prices

The first one leads to mark-ups (profits) relative to perfectly competitive markets.

The second one leads to fluctuations in these mark-ups in response to shocks.

Monetary policy cannot do anything about the first one, but can alleviate the second one.

Households

There are now many goods indexed by $i \in [0,1]$.

Households value only aggregate consumption which is assumed to be given by

$$C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

with $\epsilon > 1$.

Problem:

$$\max_{C_t(i), N_t, B_t} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{(N_t)^{1+\eta}}{1+\eta} \right)$$

subject to

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t N_t - T_t \text{ for all } t$$

IS equation

Define the aggregate price index by

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$$

Once can show (Appendix!) that the demand for individual goods is given by

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t$$

Hence, the FOCs are given by

$$\frac{C_t^\sigma}{(1-N_t)^\eta} = \frac{W_t}{P_t}$$

$$\beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^\sigma (1+i_t) \frac{P_t}{P_{t+1}} \right] = 1$$

Hence, the Euler equation – or IS equation – remains unchanged.

Firms

The production function is given by $A_t N_t(i)^\alpha$ where $\alpha \in (0,1)$.

Demand for individual goods are given by

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t = Y_t(i) = A_t N_t(i)^\alpha$$

Firms charge a mark-up $\mu_t(i)$ over their marginal costs (Appendix!)

$$\mu_t(i) = \frac{\epsilon}{\epsilon - 1} = \frac{MPL_t(i)}{W_t/P_t(i)}$$

The marginal product of labour is given by

$$MPL_t(i) = \alpha \frac{Y_t(i)}{N_t(i)}$$

Mark-ups

This yields a **mark-up** condition

$$P_t(i) = \left(\frac{\epsilon}{\epsilon - 1}\right) \varphi_t(i) \equiv \mu \varphi_t(i).$$

The mark-up μ measures the inefficiency from monopolistic competition and is a surcharge over a firm's nominal marginal costs.

It depends on how easily goods can be substituted:

- price elasticity of demand is given by $-\epsilon$
- if $\epsilon = \infty$, we get perfect competition
- if $\epsilon \to 1$, market power increases

Since marginal costs are the same across all firms, prices are also identical across firms.

Hence, without frictions inflation depends 1-1 on the price setting behaviour of firms.

APPENDIX

Demand for individual goods

How do we choose $C_t(i)$ to achieve the maximum aggregate consumption, holding fixed the total expenditure at some level Z_t?

$$\max_{C_t(i)} C_t$$

subject to

$$\int_0^1 P_t(i)C_t(i) = Z_t$$

FOC:

$$\frac{C_t(i)}{C_t(j)} = \left(\frac{P_t(i)}{P_t(j)}\right)^{-\epsilon}$$

The parameter ϵ is the elasticity of substitution between two goods.

Define the aggregate price index by

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$$

Plug in $C_t(i)$ in the expenditure constraint to get

$$C_t(j) P_t(j)^{\epsilon} \int_0^1 P_t(i)^{1-\epsilon} di = Z_t$$

$$C_t(j) = \frac{Z_t}{P_t} \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon}$$

From the definition of C_t, we have that $Z_t = P_t C_t$. Hence,

$$C_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} C_t$$

The parameter ϵ is the price elasticity of demand for any good j.

Aggregate Demand

We can aggregate across demand for individual goods to obtain

$$\begin{aligned}\left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}}di\right)^{\frac{\epsilon}{\epsilon-1}} &= C_t P_t^{\epsilon}\left(\int_0^1 P_t(i)^{1-\epsilon}di\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= C_t P_t^{\epsilon} P_t^{-\epsilon} \\ &= C_t\end{aligned}$$

Note that we have used the definition of the price index.

The household, thus, choses first aggregate demand and then divides his demand among individual goods according to the distribution of individual prices and the elasticity ϵ.

Firms – Optimal Price Setting

The production function is given by $A_t N_t(i)^\alpha$.

The nominal costs of producing output $Y_t(i)$ are thus given by

$$W_t N_t(i) = W_t \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{\alpha}}$$

Firms set prices as monopolists to maximize profits:

$$\max_{P_t(i)} P_t(i) Y_t(i) - W_t N_t(i)$$

subject to

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t$$

$$N_t(i) = \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{\alpha}}$$

FOC:

$$P_t(i)\frac{\partial Y_t(i)}{\partial P_t(i)} + Y_t(i) - W_t\frac{1}{\alpha A_t}\left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{\alpha}-1}\frac{\partial Y_t(i)}{\partial P_t(i)} = 0$$

where

$$\frac{\partial Y_t(i)}{\partial P_t(i)} = (-\epsilon)\frac{Y_t(i)}{P_t(i)}$$

since ϵ is the price elasticity of demand.

Hence:

$$P_t(i) = \left(\frac{\epsilon}{\epsilon - 1}\right)\frac{W_t N_t(i)}{\alpha Y_t(i)}$$

The last term are the nominal marginal costs when producing $Y_t(i)$.