ECON 815

A Classical Monetary Economy

Winter 2020

Firms

There is no capital. Labour productivity follows an AR(1) process

$$\log A_t = \rho \log A_{t-1} + \epsilon_t$$

where $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$.

Taking prices and nominal wages as given, firms then solve

$$\max_{N_t} P_t Y_t - W_t N_t$$

subject to

$$Y_t = A N_t^\alpha$$

where $\alpha \in (0, 1)$.

FOC:

$$\frac{W_t}{P_t} = \alpha A N_t^{\alpha - 1}$$

Households

$$\max_{C_t, N_t, B_t, M_t/P_t} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\left(\frac{M_t}{P_t}\right)^{1-\nu}}{1-\nu} + \frac{(1-N_t)^{1-\eta}}{1-\eta} \right) \right]$$

subject to

$$P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t - T_t \quad \text{for all } t$$

- T_t are **nominal** transfers (government, profits)
- $Q_t = \frac{1}{1+i_t}$ is the **nominal** bond price
- M_t is **nominal** money holdings

Using $A_t = B_t + M_t$ for total financial assets, we have

$$P_t C_t + Q_t A_t + (1-Q_t) P_t \frac{M_t}{P_t} \leq A_{t-1} + W_t N_t - T_t$$

Money Demand Equation

FOC are given by

$$
\begin{aligned}
\frac{W_t}{P_t} &= \frac{C_t^\sigma}{(1-N_t)^\eta} \\
1 &= \beta E_t \left[\frac{C_t^\sigma}{C_{t+1}^\sigma} \frac{P_t}{P_{t+1}} (1+i_t) \right] \\
\frac{M_t}{P_t} &= C_t^{\frac{\sigma}{\nu}} \left(\frac{i_t}{1+i_t} \right)^{-\frac{1}{\nu}}
\end{aligned}
$$

The last equation is money demand.

Money demand is linked to the demand for real balances which

- increase with output (income) ...
- ... and decrease with the (risk-free) nominal interest rate.

The IS Equation

We have

$$E_t\left[(1+i_t)\frac{P_t}{P_{t+1}}\right] \simeq i_t - E_t[\pi_{t+1}]$$

which yields a forward-looking **Fisher equation**

$$r_t = i_t - E_t[\pi_{t+1}].$$

The intertemporal Euler equation is approximately given by

$$\sigma E_t\left[\log\frac{C_{t+1}}{C_t}\right] = \log\beta - E_t(\pi_{t+1}) + i_t$$

or

$$E_t\left[\log\frac{C_{t+1}}{C_t}\right] = \frac{1}{\sigma}\left(r_t - \bar{r}\right)$$

so that the consumption growth rate once again depends on deviations of the (expected) real interest rate r_t from the steady state interest rate $\bar{r}$.

Classical Dichotomy

We can solve the real side independent of (i_t, π_t, M_t).

1) The IS equation determines savings/investments.

2) The marginal product of labour determines the real wage.

3) Market clearing determines output $y_t = c_t$.

We have two equations that pin down nominal variables (i_t, m_t, π_t).

1) Fisher equation $r_t = i_t - E_t[\pi_{t+1}]$

2) Money demand equation $M_t/P_t = f(i_t)$

Conclusion:

We have two dichotomous blocks for the model economy. Monetary policy either picks the money supply M_t or nominal interest rates i_t to determine only nominal variables.

Why is there money?

Money is costly. Why? Forgive interest rate for bonds, in order to hold money (rate-of-return dominance).

Hence, people would like to economize on holding real balances as much as possible.

In the background, there is some service money provides that bonds cannot.

- Cash-in-Advance constraint
- Search models
- OG models

In our model, money just happens to "show up" in the utility function.

Still, driving $i_t \to 0$ maximizes utility.

Welfare Costs of Inflation

The social marginal costs of producing real balances is zero.

But there is an opportunity cost of holding money which is the lost nominal interest rates.

This gives rise to a wedge which leads to an inefficiency.

Question: How could we measure the welfare cost associated with inflation?

1) Measure the area under the money demand function given i as a loss in consumer surplus.

2) How much are households willing to pay in percent of steady-sate consumption in order to "accept" a change in the steady-state nominal interest rate Δi?

The Friedman Rule

What would the optimal monetary policy be?

It turns out that one needs to drive i_t to zero for all t.

Suppose there are no fluctuations. Then, we have from the Fisher equation that for $i_t = 0$

$$1 + \pi = \frac{1}{1+r} = \beta$$

Prices have to decline at the discount rate or – equivalently – that there is deflation according to the rate of time preference.

This implies that there is a strictly positive return of $r = (1-\beta)/\beta$ on money.

This is the so-called **Friedman Rule**.