ECON 815

Analyzing RBC Dynamics

Winter 2020

Dynamics

Dynamics in the economy are given by a over solutionn endogenous law of motion for capital

$$k_{t+1} = g(k_t, z_t)$$

and the exogenous law of motion for productivity

$$\log z_{t+1} = \rho \log z_t + \epsilon_t.$$

There are many ways to solve for the law of motion of capital. All of these are *approximations* to the actual solution.

We will employ log-linearization to derive a (local) approximation.

Good news: `DYNARE` can log-linearize equations for us.

Good news: `DYNARE` can solve these equations for us.

Solution – An Overview

The equilibrium dynamics are described by

$$\begin{aligned}\hat{k}_{t+1} &= a_1\hat{k}_t + a_2\hat{z}_t \\ \hat{\lambda}_{t+1} &= b_1\hat{k}_t + b_2\hat{z}_t \\ \hat{z}_{t+1} &= \rho\hat{z}_t + \epsilon_t\end{aligned}$$

where λ_t is the Lagrange-multiplier which will pin down all "jump" (or decision) variables.

All variables are expressed as deviations in percent from steady state.

Solutions (`DYNARE!`) amount to finding (a_1, a_2, b_1, b_2).

Interpretation: If $\hat{k}_t = 0.01$ – a one per cent deviation from steady state – and $\hat{z}_t = 0$, then $\hat{k}_{t+1}$ will deviate by $a_1 \cdot 0.01$ per cent.

Important: The system needs to be (i) locally stable around the steady state and (ii) be a good approximation for "small" deviations.

Analysis I: Impulse Response Functions

We have a linearized law of motion for the state variables and jump variables are just functions of the state.

Consequence: IRFs are non-stochastic and can be calculated by iteration directly.

Procedure (DYNARE!):

- start out with steady-state values: $\bar{k}$ and $\log \bar{z} = 0$
- assume a one-standard deviation shock: $\epsilon_0 = \sigma_\epsilon$
- calculate from the law of motion, $\{\hat{k}_{t+1}\}_t$ and $\{\hat{\lambda}_t\}_t$
- use these values to calculate series for all other variables

More generally, one can work with a one-time deviation from a sequence of shocks.

Analysis II: Simulations

We generate N samples of length T (`DYNARE!`).

- generate M random draws for the productivity shocks
- simulate the linearized economy (all variables of interest) with these shocks
- trim the sample by the first $M - T$ observations
- do this N times

Detrend (if necessary) the simulated data as with the real data.

Then, compute sample moments (`DYNARE!`) such as variances, covariances, autocorrelations, etc. from the simulated data as an average across the N samples.

Compare these with the data, including standard errors for the simulated moments.

RBC Model

We can write our RBC model as

$$\begin{aligned} \theta(1-n_t)^{-\eta} &= (1-\alpha)\frac{y_t}{n_t}c_t^{-\gamma} \\ c_t^{-\gamma} &= \beta E_t\left[c_{t+1}^{-\gamma}\left(\alpha\frac{y_{t+1}}{k_{t+1}}+(1-\delta)\right)\right] \\ c_t+k_{t+1} &= y_t+(1-\delta)k_t \\ y_t &= z_t k_{t+1}^{\alpha} n_t^{1-\alpha} \\ \log z_t &= \rho\log z_{t-1}+\epsilon_t \end{aligned}$$

We have four endogenous variables we want to look at:

- output y
- labour supply n
- consumption c
- capital k

Model in DYNARE – Logs

Some equations:

$$\texttt{theta} * (1 - \texttt{exp(lab)})\hat{\ }(-\texttt{eta}) = \ldots;$$
$$\texttt{exp(c)}\hat{\ }(-\texttt{gam}) = \texttt{bet} * \texttt{exp(c(+1))}\hat{\ }(-\texttt{gam}) * \ldots;$$
$$\texttt{exp(c)} + \texttt{exp(k)} = \texttt{exp(y)} + (1 - \texttt{delta}) * \texttt{exp(k(-1))};$$
$$\texttt{exp(y)} = \texttt{exp(z)} + \texttt{alpha} * \texttt{exp(k(-1))}\hat{\ }\alpha * \texttt{exp(lab)}\hat{\ }(1 - \texttt{alpha});$$
$$\texttt{z} = \texttt{rho} * \texttt{z}(-1) + \texttt{e};$$

Important issues:

- model will treat variables in logs – `z` is in logs already, `c` is $\log c$
- decision (jump) variables have current index
- state variables in period t have index -1
  - $\texttt{k}(-1)$ is capital just before period t
  - $\texttt{z}(-1)$ is productivity just before shock in period t
- forward looking variables are in expectations and indexed $+1$

Policy Functions

The solution of the linearized model is in percentage deviations from steady state.

$$k_{t+1} = \bar{k} + a_1(k_t - \bar{k}) + a_2(z_t - \bar{z})$$

The interpretation in `DYNARE` of the coefficients is according to

$$\mathtt{k} = \bar{\mathtt{k}} + \mathtt{a}_1(\mathtt{k}(-1) - \bar{\mathtt{k}}) + \mathtt{a}_2(\mathtt{z}(-1) - \bar{\mathtt{z}}) + \mathtt{a}_3\mathtt{e}.$$

so that $\mathtt{a}_1 = a_1$, $\mathtt{a}_2 = \rho a_2$ and $\mathtt{a}_3 = a_2$ with $\bar{z} = 0$.

Hence, the solution distinguishes between the effects of the current shock and the lagged term from the AR(1) shock process.

Beyond this, `DYNARE` delivers

- impulse response functions
- autocorrelations
- second order moments

IRFs

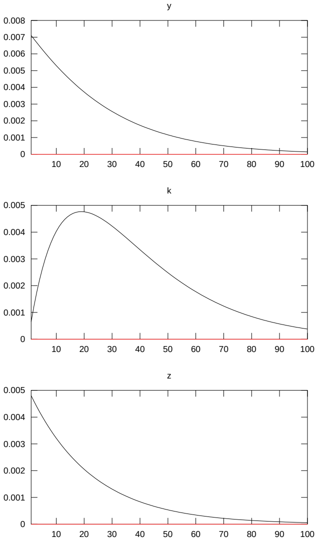

Figure: Baseline model – Canadian Data

Correlations

MATRIX OF CORRELATIONS

Variables	y	c	k	lab	z
y	1.0000	0.9220	0.8357	0.6758	0.9950
c	0.9220	1.0000	0.9832	0.3377	0.8788
k	0.8357	0.9832	1.0000	0.1599	0.7767
lab	0.6758	0.3377	0.1599	1.0000	0.7460
z	0.9950	0.8788	0.7767	0.7460	1.0000

COEFFICIENTS OF AUTOCORRELATION

Order	1	2	3	4	5
y	0.9659	0.9327	0.9005	0.8691	0.8387
c	0.9943	0.9866	0.9772	0.9661	0.9536
k	0.9986	0.9948	0.9888	0.9808	0.9710
lab	0.8999	0.8075	0.7224	0.6439	0.5716
z	0.9562	0.9143	0.8743	0.8360	0.7994

Match with Data

Golden ratios?

- $c/y = 0.79$ and $k/y = 8.26$ from calibration

Covariance with output?

- labour – 0.67 vs. 0.7 (data); consumption – 0.92 vs. 0.53 (data)

`Standard deviations`

Variables	SD	% of SD(y)
y	0.0275	1
c	0.0215	0.78
k	0.0284	1.03
lab	0.0079	0.28
z	0.0164	0.60

Compared to the data?

- labour/output – 28% vs. 97% (data)
- consumption/output – 78% vs. 72% (data)
- output – model explains 75% of SD in the data (0.0367)

Critique – No Amplification

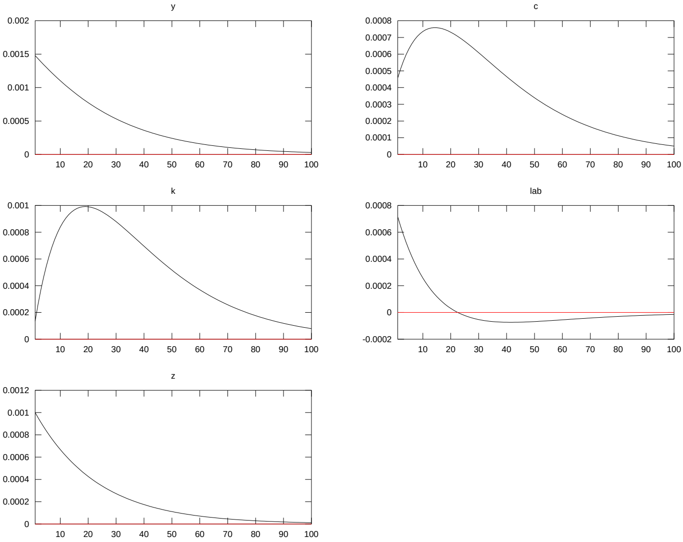

Figure: Low shocks – Canadian Data

Critique – No Propagation

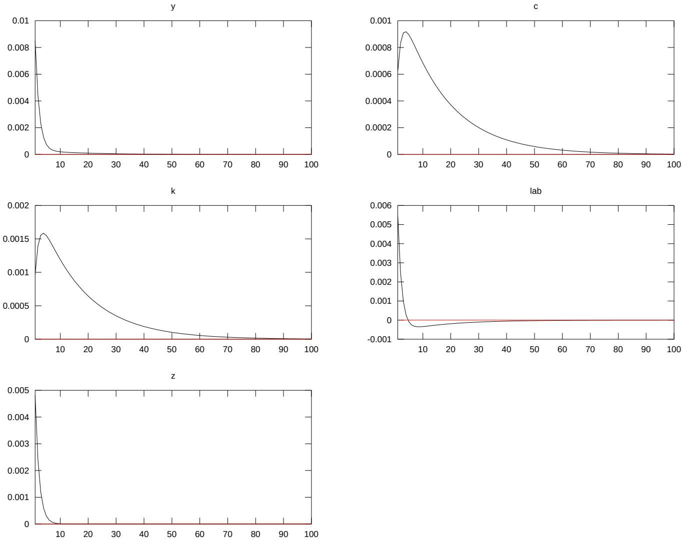

Figure: Low autocorrelation – Canadian Data

Critique – Labour Supply Elasticity

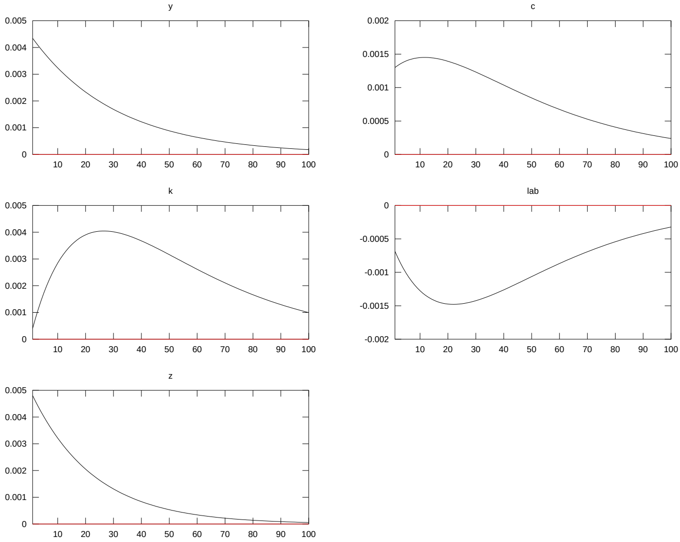

Figure: Model for $\gamma = \eta = 5$

Labour Elasticity (cont.)

We have in the model

$$\begin{aligned} l_t + n_t &= 1 \\ \theta l_t^{-\eta} &= \lambda(z_t) w_t \end{aligned}$$

where l_t is leisure and η is the Frisch elasticity of labour supply.

Log-linearizing, we obtain

$$\hat{n}_t = \frac{\bar{l}}{\bar{n}} \frac{1}{\eta} \left(\hat{w}_t + \hat{\lambda}_t \right)$$

For $\eta = 1$ and $\bar{l} = 0.8$, we obtain a huge response of labour supply to changes in wages. Empirically, this is not the case.

The model fits best for the linear labour model which we interpret as (efficient!) changes in (un)employment.

Summary

Ken Rogoff:

"The real business cycle results..., are certainly productive. It has been said that a brilliant theory is one which at first seems ridiculous and later seems obvious. There are many that feel that (RBC) research has passed the first test. But they should recognize the definite possibility that it may someday pass the second test as well."

My assessment:

It shows the power of the DSGE approach. But it would be foolish to think that business cycles are entirely driven by highly persistent technology shocks and the large – and efficient – reaction of labor input to such shocks.

The RBC model is particularly "vulnerable" to changes in the Frisch elasticity of labour supply. But one can argue that this is precisely what we are most interested in (employment responses).