Economics 442 **Winter 2015**

Topics in Public Economics **Thorsten Koeppl**

Assignment 4

(Due: Tuesday, April 7 – Drop Box until 3:00 pm)

1. Consider again an economy with log-utility, $\beta = 1$ and a Cobb-Douglas production function with full depreciation of capital. There is a PAYG system in steady state given by

$$\tau(k) = \frac{(1-\alpha)Ak^{\alpha} - nk}{1 + \frac{n}{\alpha A}k^{1-\alpha}}$$

Assume further that α is sufficiently small, so that without the PAYG system the economy would have too much capital, i.e.

$$\bar{k} > k_{GR} = \left(\frac{\alpha A}{n}\right)^{\frac{1}{1-\alpha}}.$$

(a) Find the optimal PAYG τ that achieves k_{GR} in steady state.

(b) How does τ change when n falls? How does it change when A falls?

Set the remaining parameters now to $A = 1$ and $n = 1$. Consider (i) a fall in n and (ii) a fall in A.

(c) Do pension $a = \tau n$ in steady state decrease or increase for these two cases? What is the intuition for your result?

2. Consider an overlapping generations economy where people can invest in the education of their children. Their preferences are given by

$$\ln c_t(t) + \beta \ln c_t(t+1) + \gamma \ln e(t)$$

where $e(t)$ is investment of generation t in education. Education investment is productive and yields human capital next period according to

$$h(t+1) = Be(t)^{\theta}h(t)^{1-\theta}.$$

with $h(0)$ being the initial endowment of human capital for generation 0. People have a total effective endowment of labour equal to $h(t)$ which they supply at a wage rate $w(t)$. Their total income is given by $w(t)h(t)$.

Production takes place according to the production function given by

$$F(K_t, L_t) = AK_t^{\alpha}L_t^{1-\alpha}$$

where $L_t = h(t)$. The per-capita capital stock is now expressed as $k_t = K_t/L_t$.

(a) Find the law of motion for human capital in terms of wages $w(t)$.

(b) Find the law of motion for the per-capita capital stock.

(c) Find the steady state per-capita capital stock.

(d) What is the long-run growth rate of human capital?

Suppose now that there are two households, academics and non-academics. The former has an initial endowment of human capital given by $h_a(0) = 2$, while the latter has an endowment equal to $h_0(0) = 1$. For parameters, set $\beta = 1$, $A = 1$ and $B = 1$.

(e) Find the steady state per-capita capital stock and wage rate. [Hint: You can use directly the law of motion you found earlier, simply distinguishing between $s_a(t+1)$ and $s_0(t+1)$ taking into account that $L_{t+1} = h_a(t+1) + h_0(t+1)$.]

(f) How is long-run inequality in human capital and consumption related to the initial endowment?

Suppose the economy starts out in steady state you have found above, but one redistributes income in the first period only according to $\bar{w}h_a(0)(1 - \tau_a)$ and $\bar{w}h_0(0)(1 + \tau_0)$.

(g) When is such a policy feasible?

(h) Can such a policy achieve the same level of income, consumption and human capital across households in steady state? If so, what is level of taxes (τ_a, τ_0)?

3. For this question, please refer to the paper by Krusell and Smith, "Is Piketty's Law of Capitalism Fundamental".

   The "gross model" has an exogenously given savings rate s which determines the fraction of gross income saved. Hence, $s = i_t/y_t$.

   The "net model" has an exogenously given savings rate $\tilde{s}$ which determines the fraction of net income saved. Hence, $\tilde{s} = \tilde{\imath}_t/\tilde{y}_t = (i_t - \delta k_t)/(y_t - \delta k_t)$

   (a) Calculate the following expressions for the "gross model" in steady state: (i) capital/income ratio; (ii) capital/net income ratio; (iii) net savings rate.

   (b) Calculate the following expressions for the "net model" in steady state: (i) capital/income ratio; (ii) capital/net income ratio; (iii) gross savings rate.

   Set now $\delta = 0.05$ and consider two growth rates $g = 0.03$ and $g = 0.01$.

   (c) Set $s = 0.2$. What happens to the capital/income, capital/net income ratio and the net savings rate in the gross model?

   (d) Set $\tilde{s} = 0.2$. What happens to the capital/income, capital/net income ratio and the gross savings rate in the net model?