Economics 442 **Winter 2015**

Topics in Public Economics **Thorsten Koeppl**

Assignment 3

(Due: Friday, March 20 – Drop Box until 3:00 pm)

1. Consider an environment with OG where the population size grows over time at rate n and where the size of the initial old generation is normalized to $N_{-1} = 1$. Each generation has an endowment of one unit of labor when young and none when old. The initial old have an endowment of capital given by k_0. Preferences for each generation t are given by

$$u(c_t(t), c_t(t+1)) = \ln c_t(t) + \beta \ln c_t(t+1),$$

with preferences for the initial old given by $u(c_{-1}(0)) = \ln c_{-1}(0)$. The aggregate production function in this economy is given by

$$Y(t) = AN_t^{1-\alpha}K(t)^{\alpha}$$

with full depreciation of capital each period. Assume that $\alpha = 1/4$ and $\beta = 1$.

(a) Find the steady state level of capital $\bar{k}$ and the level of capital in the golden rule allocation k_{GR}. Is there over- or underaccumulation of capital relative to the golden rule? [Hint: You may use the formulas that you have derived in Question 1 of Assignment 3.]

Consider a PAYG scheme with taxes τ on the young generation and pay-outs a to the old generation.

(b) Find the savings function given the PAYG system.

(c) What levels of τ and a achieve the golden rule allocation in part (a)?

2. Consider again the economy from Question 1. Households, however, are now *myopic*, i.e. they consume an exogenously given fraction of their first period income. In particular, assume that their consumption when young is given exogenously by

$$c_t(t) = \frac{1}{1+\rho} w(t)$$

where $w(t)$ is their first period wage income and $\rho < \beta$.

(a) Find the new law of motion on the per capita capital stock. [Hint: The household does not optimize and his savings function is now directly determined for any value of $k(t)$ by his first period budget constraint.]

(b) Find the steady state level and the associated interest rates.

Assume that $\rho = 1/2$, $\beta = 1$, $\alpha = 1/4$, $n = 1$ and $A = 4$.

(c) Find *all* social security systems (τ, k^s) such that the steady state with myopic households corresponds to the golden rule allocation from Question 1.

(d) Can one achieve the golden rule allocation with a PAYG scheme? What about a fully funded pension scheme?

3. Consider a steady state with a PAYG pension system given by

$$\tau(k) = \frac{\beta(1-\alpha)Ak^{\alpha} - (1+\beta)nk}{\beta + \frac{n}{\alpha A}k^{1-\alpha}}$$

Set parameters to $\beta = 1$, $\alpha = 1/2$, $A = 2$ and $n = 1$.

(a) Draw the graph $\tau(k)$. For which values of k is the PAYG system given by $\tau = a = 0$?

(b) Solve for the maximal transfer τ that can be sustained with a PAYG system.

(c) Let $n = 1/2$. Find the PAYG system that maintains a steady state capital stock equal to $\bar{k} = 1/4$. Explain why a PAYG system has now higher transfers τ than in part (a) for this capital stock.

(d) Let $n = 2$. Can we still achieve a PAYG system with a steady state level of capital given by $\bar{k} = 1/4$? Explain your answer.