Economics 442 **Winter 2015**

Topics in Public Economics **Thorsten Koeppl**

Assignment 2

(Due: Friday, February 13: Drop Box until 3:00 pm)

1. Consider the neoclassical growth model with OG where the population size grows over time at rate n and where the size of the initial old generation is normalized to $N_{-1} = 1$. Each generation has an endowment of one unit of labor when young and none when old. The initial old have an endowment of capital given by k_0. Preferences for each generation t are given by

$$u(c_t(t), c_t(t+1)) = \ln c_t(t) + \beta \ln c_t(t+1),$$

with preferences for the initial old given by $u(c_{-1}(0)) = \ln c_{-1}(0)$. The aggregate production function in this economy is given by

$$Y(t) = AN_t^{1-\alpha} K(t)^{\alpha}$$

with full depreciation of capital each period.

(a) Derive the law of motion for the per capita capital stock, i.e. derive $k(t+1)$ as a function of parameters and $k(t)$.

(b) What is the steady state per capita capital stock and the steady state interest rate?

(c) Find the golden rule per capita capital stock and consumption allocation.

(d) At what levels of parameters is there over- and underaccumulation in the steady state?

Assume now $\alpha = 1/2$, $\beta = 1$, $n = 1$ and $A = 4$. Set up an excel spreadsheet to do the following computations for the first 30 generations.

(e) Using the law of motion you have found in part (a), find and plot the per capita capital stock $k(t+1)$, consumption allocation $(c_t(t), c_t(t+1))$ and utility of the generations over time for (i) $k(0) = 0.5$ and (ii) $k(0) = 1.5$.

2. Consider again the economy from Question 1. Suppose now that there is a government that roles over a constant level of per capita debt

$$\frac{B(t)}{N_{t-1}} = b(t) = b$$

and levies a lump-sum tax or transfer on the young. The government, however, does not consume anything ($G(t) = 0$) or tax the old ($\tau_2 = 0$).

(a) Write down the government budget constraint in per capita terms for any arbitrary period t and for the steady state.

(b) Households can now save in two assets, capital $K(t+1)$ and bonds $B(t+1)$. In equilibrium aggregate savings have to equal total assets. Write down the relationship between aggregate savings and total assets in per capital terms for any arbitrary period t and for the steady state. [Hint: Derive the demand for savings as a function of total first period income, $w(t) - \tau_1(t)$.]

(c) Find the new steady state per capita level for capital as a function of per capita debt $b(t)$. For what values of debt does the economy achieve the golden rule capital stock k_{GR}? [Hint: Use the results of part (a) and (b) to obtain two equations in the three unknowns (k, b, τ_1). Then, use the value of k_{GR} from the previous question.]

3. Consider once again the economy of Question 1. The size of the initial old generation is normalized to $N_{-1} = 1$ and set $A = 1$ and $\beta = 1$. Note that α and n are free parameters. Suppose the economy is in steady state initially with capital $\bar{k}$ and there is a natural disaster that wipes out 50% of the capital stock. Hence, the economy starts out with $\bar{k}/2$ in Period $t = 0$. In response to the disaster, the government imposes a transfer scheme $\tau_1(0)$ and $\tau_2(0)$ in Period 0.

(a) Suppose the government wants to ensure that the initial old generation's consumption $c_{-1}(0)$ stays the same as without the disaster. How large does the transfer $\tau_2(0)$ have to be?

(b) How large is the tax $\tau_1(0)$ that is required from each member of the young generation?

(c) Set $n = 1$. For which values of α is the transfer scheme feasible? Discuss how your answer would change as you increase n and interpret your finding. [Hint: You cannot solve for α in closed form.]

Suppose now there is a government that does not care about the old generation at all and wants to recover from the desaster as quick as possible. To do so, it imposes a tax on the old generation and makes a transfer to the young generation. Set $\alpha = 1/2$.

(d) Find the maximum tax that can be imposed on the old generation.

(e) Find the savings of a member of the new generation with the transfer.

(f) Set $n = 1$. For which values of α can the economy get back to its old steady state capital stock in period 1? Discuss how your answer would change as you increase n and interpret your finding. [Hint: You cannot solve for α in closed form.]