Economics 442 **Winter 2015**

Topics in Public Economics **Thorsten Koeppl**

Assignment 1

(Due: Thursday, January 29 – Drop Box by 3pm)

1. Each period a generation with size N_t is born, where the population grows (shrinks) at a constant rate $n > 1$ ($n < 1$). All generations have an endowment of y of the single consumption good when young and no endowment when old. Preferences for each generation t are given by

$$u(c_t(t), c_t(t+1)) = \sqrt{c_t(t)} + \sqrt{c_t(t+1)},$$

with preferences for the initial old given by $u(c_{-1}(0)) = \sqrt{c_{-1}(0)}$.

   (a) Derive the set of feasible allocations for this economy in terms of the population growth rate n. Draw a diagram properly labeled showing the feasible set.

   Set now $n = 2$, i.e. the population doubles each generation.

   (b) Show that the stationary allocation $(c_1, c_2) = (\frac{3}{4}y, \frac{1}{2}y)$ is **not** Pareto optimal. [Hint: Find a feasible allocation that pareto-dominates this allocation.]

   (c) Set $y = 1$. Find **all** stationary Pareto optimal allocations for this economy.

   (d) Find a stationary transfer scheme (τ_1, τ_2) that achieves the best allocation for all generations except the initial old.

   Suppose now that the young have access to a storage technology that yields a gross return $r > 0$; i.e. if 1 unit of the good is invested today, it yields r units tomorrow.

   (e) In the absence of transfers, find the optimal storage of resources by each generation. For which values of r do all generations except the initial old prefer storage over the transfer scheme that you have found in part (d)?

   (f) Suppose the transfer scheme lasts only for T periods. How does your anwser to part (e) change, if this is publicly announced? What if noone anticipates that the scheme ends at T? Explain your answer.

2. Consider an OG environment with preferences equal to

$$u(c_t(t), c_t(t+1)) = \ln c_t(t) + \ln c_t(t+1),$$

for all generations except the initial old whose preferences are given by $u(c_{-1}(0)) = \ln c_{-1}(0)$. Assume that the population doubles over time, i.e. $n = 2$ and normalize $N_{-1} = 1$. Each member of a generation has an endowment of $y_1 = 2$ when young and $y_2 = 1$ when old. There is an initial stock of debt equal to b_{-1} owned by the initial old generation.

(a) Write down the balanced budget condition for the government using the interest rate $r(t)$ on debt it issues in period t.

(b) Set up the household's decision problem and find the FOC in terms of the interest rate $r(t)$ on debt.

(c) Find a stationary (in consumption) perfect foresight equilibrium. What is the initial amount of debt b_{-1}, the sequence of debt levels and the sequence of interest rates associated with this equilibrium?

(d) Find an equivalent lump-sum tax scheme that yields the same stationary perfect foresight equilibrium.

(e) Use an excel program to plot the evolution of debt and interest rates when $b_{-1} = 1.01 b_{-1}^{SS}$. Is such a debt policy feasible? [Hint: Use the algorithm described in class.]

(f) Plot now a graph for the evolution of debt, interest rates and consumption allocation when $b_{-1} = 0.99 b_{-1}^{SS}$. Does such a policy enhance welfare for all generations?

3. Consider again an OG environment with preferences as given in the previous question. The endowment across generations is now given by $y_1 = y_2 = 1$. The size of the intial old generation is again normalized, $N_{-1} = 1$, and there is population growth equal to $n = 2$.

   (a) Find the optimal allocation for this economy neglecting the initial old generation. [Hint: Solve the Pareto-problem in class.]

   Suppose that there are no taxes and no debt in this economy. A new government makes the following proposal: issue an amount of debt b_0, use the proceeds $b_0/(1+r)$ to make a one-time transfer to the initial old and then role over the debt forever. The government also promises to keep the per-capita debt level constant over time.

   (b) If the government needs a majority to vote for the proposal, will it be adopted? [Hint: Find the stationary equilibrium with debt starting in period 0 and compare the utility for all generations of the equilibrium with the utility under autarky.]

   Suppose now that the government instead proposes to issue debt b_0, keep this level of debt constant over time, but also to tax old people τ_2 to finance the building of useless pyramids. The revenue from the tax is given by $\tau_2 N_{t-1}$ so that overall consumption is given by $N_t g = \tau_2 N_{t-1}$. The government also specifies that the proceeds of issuing debt will be used as a one-time transfer for the construction of a statue.

   (c) Set up the government budget constraint and derive the households net-present value budget constraint. [Hint: You may neglect the special nature of the tansfer arising from the initial debt issuance.]

   (d) Show that for a stationary equilibrium we still need $(1 + r) = n$. [Hint: The total resources available for private consumption are now $N_t y_1 + N_{t-1}(y_2 - \tau_2)$.]

   (e) Find the stationary equilibrium given by c_1, c_2 and b_0 in terms of the tax τ_2.

   (f) How large can the government set its tax τ_2, if it needs a majority to vote for its policy?

   (g) Can the policy be implemented, if the government needs a 2/3 plus one vote majority? Explain your answer.