ECON 442

Intergenerational Smoothing

Winter 2015

Thought Experiment

Growth model:

- log preferences and no discounting ($\beta = 1$)
- constant-returns-to-scale production
- The economy is in steady state with capital stock $\bar{k}$.

There is a natural disaster, so that half of the capital stock is destroyed

$$k(0) = \frac{\bar{k}}{2}$$

What's the best path over time to get the economy back to its steady state?

How can a government ensure this?

Equilibrium transition

Output and labor income for the young drop to

$$\begin{aligned} y(0) &= A\left(\frac{\bar{k}}{2}\right)^{\alpha} \\ w(0) &= (1-\alpha)A\left(\frac{\bar{k}}{2}\right)^{\alpha} \end{aligned}$$

Hence, for capital accumulation we obtain the following sequence

$$\begin{aligned} k(1) &= \frac{1}{2n}w(0) = \frac{1}{2n}(1-\alpha)A\left(\frac{\bar{k}}{2}\right)^{\alpha} = \kappa\left(\frac{\bar{k}}{2}\right)^{\alpha} \\ k(2) &= \frac{1}{2n}w(1) = \frac{1}{2n}(1-\alpha)A\left(\frac{k(1)}{2}\right)^{\alpha} = \kappa\left(k(1)\right)^{\alpha} = \kappa\left(\kappa\left(\frac{\bar{k}}{2}\right)^{\alpha}\right)^{\alpha} \\ &= \kappa^{1+\alpha}\left(\frac{\bar{k}}{2}\right)^{\alpha^2} \\ k(3) &= \kappa^{1+\alpha+\alpha^2}\left(\frac{\bar{k}}{2}\right)^{\alpha^3} \qquad \text{and so on ...} \end{aligned}$$

Capital evolves over time according to

$$k(t+1) = \kappa^{\sum_{s=0}^{t} \alpha^s} \left(\frac{\bar{k}}{2}\right)^{\alpha^{t+1}}.$$

The economy converges back to its old steady state $\bar{k} = \kappa^{\frac{1}{1-\alpha}}$.

However, the impact of the disaster is felt more by earlier generations.

The initial old lose

$$\Delta u_{-1} = \ln\left(\alpha A \left(\frac{\bar{k}}{2}\right)^{\alpha}\right) - \ln(\alpha A \bar{k}^{\alpha}) = \ln(1/2)^{\alpha} < 0$$

All other generations lose

$$\Delta u_t = \ln\left(\frac{k(t)k(t+1)}{\bar{k}^2}\right)^{\alpha} \longrightarrow 0$$

since $k(t)$ increases over time and converges to $\bar{k}$.

Intergenerational Conflict

Case 1: Maintain the initial old's current consumption.

We need to tax the young people in period 0

$$N_0\tau_1(0) = N_{-1}n\alpha A\left(\bar{k}^\alpha - \left(\frac{\bar{k}}{2}\right)^\alpha\right)\left(= -N_{-1}\tau_2(0)\right)$$

After taxes, the young now have income

$$\begin{aligned} w(0) - \tau_1(0) &= (1-\alpha)A\left(\frac{\bar{k}}{2}\right)^\alpha - \alpha A\left(\bar{k}^\alpha - \left(\frac{\bar{k}}{2}\right)^\alpha\right) \\ &= A\bar{k}^\alpha\left[\left(\frac{1}{2}\right)^\alpha - \alpha\right] \end{aligned}$$

which is still positive for small enough α.

This reduces their wage income for generation 0 and hence their savings. Capital accumulation starts from a lower $k(1)$ and *all other generations* are worse off.

Case 2: Get back to the old capital level as fast as possible.

We tax the old people

$$N_{-1}\tau_2(0) = N_{-1}n\alpha A\left(\frac{\bar{k}}{2}\right)^{\alpha}\Big(= -N_0\tau_1(0)\Big)$$

The income of the young is now given by total output

$$w(0) - \tau_1(0) = (1-\alpha)A\left(\frac{\bar{k}}{2}\right)^{\alpha} + \alpha A\left(\frac{\bar{k}}{2}\right)^{\alpha} = A\left(\frac{\bar{k}}{2}\right)^{\alpha}$$

with investment equal to

$$k(1) = \frac{1}{2n}A\left(\frac{\bar{k}}{2}\right)^{\alpha}.$$

Hence, investment is higher and we converge faster to the old steady state.

Intergenerational Smoothing

Consider now the following Social Welfare Function

$$\mathcal{W} = u(c_{-1}(0))\gamma^{-1} + \sum_{t=0}^{\infty} \gamma^t \left(u(c_t(t)) + \beta u(c_t(t+1))\right)$$

or equivalently

$$\sum_{t=0}^{\infty} \gamma^t \left(u(c_t(t)) + \frac{\beta}{\gamma} u(c_{t-1}(t))\right)$$

There is a (social) discount factor γ which puts different weights on generations.

Why?

- bequest motives within families
- political economy considerations
- normative arguments?

Our discussion has covered the values $\gamma \to 0$ and $\gamma \to 1$.

Social Planner solves

$$\max_{k(t),c_t(t),c_{t-1}(t)} \sum_{t=0}^{\infty} \gamma^t \left(u(c_t(t)) + \frac{\beta}{\gamma} u(c_{t-1}(t)) \right)$$

subject to

$$Ak(t)^\alpha = nk(t+1) + c_t(t) + \frac{1}{n} c_{t-1}(t)$$

Or, equivalently,

$$\max_{k(t+1),c_t(t)} \sum_{t=0}^{\infty} \gamma^t \left(u(c_t(t)) + \frac{\beta}{\gamma} u(nAk(t)^\alpha - n^2 k(t+1) - nc_t(t)) \right)$$

FOC:

$$\begin{aligned} \frac{u'(c_t(t))}{u'(c_{t-1}(t))} &= \frac{n\beta}{\gamma} \\ \frac{u'(c_{t-1}(t))}{u'(c_t(t+1))} &= \frac{\gamma}{n} \alpha A k(t)^{\alpha-1} \end{aligned}$$

Results:

Planner does not "distort" the intertemporal allocation **within** each generation.

$$\frac{u'(c_t(t))}{\beta u'(c_t(t+1))} = \alpha A k(t)^{\alpha-1}$$

But the planner "distorts" the allocation **across** generations where

$$\alpha A k(t)^{\alpha-1} \frac{\gamma}{n} = \frac{c_t(t+1)}{c_{t-1}(t)}$$

with log-utility.

We obtain the **modified golden rule** $f'(k) = n/\gamma$.

Special case is $\gamma = 1$, where we have a representative generation.