ECON 442

Public Debt

Winter 2015

Gov't Debt

- interest-bearing asset $B(t)$
- discount bond with face value $B(t)$
- (net) real interest $r(t)$ determined in equilibrium
- HH lend $b(t)/(1+r(t))$ in t to get $b(t)$ in $t+1$

Taxes in period t:

- lump sum
- $\tau_1(t)$ on young
- $\tau_2(t)$ on old

Gov't policy is given by $(B(t), \tau_1(t), \tau_2(t))$.

Gov't Budget Constraint

Accounting relationship: Expenditure = Revenue

$$B(t-1) = \frac{B(t)}{1+r(t)} + N_t \tau_1(t) + N_{t-1} \tau_2(t)$$

In per capita terms:

$$\frac{1}{n} b(t-1) = \frac{b(t)}{1+r(t)} + \tau_1(t) + \frac{1}{n} \tau_2(t)$$

A gov't policy is feasible, if it satisfies this budget constraint.

Savings Functions

Generation t's problem when old is $c_t(t+1) = y_2 + b(t) - \tau_2(t+1)$.

Generation t's problem when young:

$$\max_{c_t(t), c_t(t+1), b(t)} u(c_t(t), c_t(t+1))$$

subject to

$$c_t(t) + \frac{b(t)}{1+r(t)} = y_1 - \tau_1(t)$$

$$c_t(t+1) = y_2 + b(t) - \tau_2(t+1)$$

Take current taxes, current interest rates and expected future taxes $\tau_2(t+1)$ as given.

Gov't bond allows the young generation to save.

Solution:

$$
\begin{aligned}
\frac{u'(c_t(t))}{\beta u'(c_t(t+1))} &= 1 + r(t) \\
c_t(t) + \frac{c_t(t+1)}{1+r(t)} &= y_1 + \frac{1}{1+r(t)} y_2 + \tau_1(t) + \frac{1}{1+r(t)} \tau_2(t+1) \equiv y_\tau
\end{aligned}
$$

Savings function is given by

$$b(t) = s((1 + r(t), y_\tau))$$

What matters is the interest rate and the (expected) NPV of (after-tax) life-time wealth y_τ.

Definition of Equilibrium

Definition: A (rational expectations) equilibrium is given by a sequence of allocations $(c_t(t), c_t(t+1), b(t))$ and interest rates $r(t)$ such that for a given level of gov't policy $(B(t), \tau_1(t), \tau_2(t))$

(i) households maximize their utility taking interest rates and the gov't policy as given

(ii) markets clear, i.e

$$\begin{aligned} N_t c_t(t) + N_{t-1} c_{t-1}(t) &= N_t y_t(t) + N_{t-1} y_{t-1}(t) \\ N_t b(t) &= B(t) \end{aligned}$$

Case 1 – No Debt

Suppose there is no debt, $b(t) = 0$ for all t

$$\tau_1(t) + \frac{1}{n}\tau_2(t) = 0$$

Recall the budget constraints:

$$\begin{aligned} c_t(t) &= y_1 + \tau_1(t) \\ c_{t-1}(t) &= y_2 + \tau_2(t) \end{aligned}$$

Adding these constraints we obtain

$$\begin{aligned} N_t c_t(t) + N_{t-1} c_{t-1}(t) &= N_t(y_1 + \tau_1(t)) + N_{t-1}(y_2 + \tau_2(t)) \\ c_t(t) + \frac{1}{n} c_{t-1}(t) &= y_1 + \frac{1}{n} y_2 \end{aligned}$$

We can achieve any allocation via lump-sum taxes.

Case 2 – Only Debt

Suppose there are no lump-sum taxes

$$\begin{aligned} N_t \frac{b(t)}{1+r(t)} &= N_{t-1} b(t-1) \\ \frac{1}{1+r(t)} b(t) &= \frac{1}{n} b(t-1) \end{aligned}$$

In a stationary equilibrium, we have

$$\begin{aligned} \frac{u'(c_1)}{\beta u'(c_2)} &= 1+r(t) \\ c_1 + \frac{1}{1+r(t)} c_2 &= y_1 + \frac{1}{1+r(t)} y_2 \\ c_1 + \frac{1}{n} c_2 &= y_1 + \frac{1}{n} y_2 \end{aligned}$$

Hence: In equilibrium $1 + r(t) = n$. Interesting!

This requires a constant level of debt, given by $c_2 = y_2 + b^*$.

Non-stationary Equilibria

Consider some arbitrary initial level of debt $b(-1)$.

Step 1: This pins down

$$c_{-1}(0) = y_2 + b(-1)$$

Step 2: Market clearing gives $c_0(0)$

$$c_0(0) + \frac{1}{n}c_{-1}(0) = y_1 + \frac{1}{n}y_2$$

Step 3: Balanced budget yields

$$\frac{1}{n}b(-1) = \frac{b(0)}{1 + r(0)}$$

Step 4: The household problem now yields two equations in two unknowns $c_0(1)$ and $r(0)$.

- Budget constraint when young

$$\begin{aligned} c_0(1) &= y_2 + b(0) \\ &= y_2 + \left(\frac{1+r(0)}{n}\right) b(-1) \end{aligned}$$

- Optimal choice of household is given by

$$\frac{u'(c_0(0))}{\beta u'(c_0(1))} = 1 + r(0)$$

Step 5: Calculate $b(0) = b(-1)\frac{1+r(0)}{n}$

Step 6: Iterate over time.

There are three possible scenarios.

- For some initial level of debt, $b(-1) = b^*$, interest rates are constant at $1 + r = n$ and debt is stationary over time.
- For $b(-1) < b^*$, debt is decreasing over time and so are interest rates.
- For $b(-1) > b^*$, debt explodes and so do interest rates.

In the last case, there is a corner solution such that $c_t(t) = y_1$ and $c_t(t+1) = y_2$ for all t.

Why? Eventually households would not be willing to hold the total stock of debt and the incentive to buy debt would unravel backwards.