ECON 442

Inefficient Equilibrium

Queen's University

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Different Trading Environments

1) Market to exchange goods against goods

2) Storage

3) Transfers

Utility function:

$$\lambda u(c_t(t)) + (1 - \lambda)u(c_t(t + 1))$$

where $\lambda \in [0, 1]$

- We are interested primarily in the case $\lambda \to 0$, i.e. households are very patient.

- We will show that there is a fundamental inefficiency and that intergenerational redistribution matters.

Env. I – Goods Market

People are price takers and exchange goods so as to maximize their utility.

There cannot be any trade.

- young are identical and old are identical
- intertemporal trade is impossible

If λ is sufficiently low, autarky is not pareto-optimal.

Let's look at a (silly) equilibrium where people make decentralized decisions about trading.

Individual decision problem:

$$\max_{c_t(t),c_t(t+1)} \lambda u(c_t(t)) + (1-\lambda)u(c_t(t+1))$$

subject to

$$p(t)c_t(t) + a(t) \le p(t)y_1$$

$$p(t+1)c_t(t+1) \le p(t+1)y_2 + (1+i_t)a(t)$$

where $a(t)$ is lending (if > 0) or borrowing (if < 0) in nominal terms.

The optimal solution satisfies the constraints and the FOC

$$\left(\frac{\lambda}{1-\lambda}\right)\frac{u'(c_t(t))}{u'(c_t(t+1))} = \frac{p(t)}{p(t+1)}(1+i_t) = 1 + r_t$$

Definition: A (rational expectations) equilibrium is given by a sequence of allocations $\{(c_t(t), c_t(t+1)), a(t)\}$ and prices $\{p(t), i(t)\}$ such that

(i) household maximize their utility taking prices as given
(ii) markets clear.

Here, market clearing requires autarky

$$\begin{aligned} c_t(t) &= y_1 \\ c_{t-1}(t) &= y_2 \\ a(t) &= 0 \end{aligned}$$

Hence:

$$\left(\frac{\lambda}{1-\lambda}\right)\frac{u'(y_1)}{u'(y_2)} = 1 + r$$

For $\lambda \to 0$, we get $r = -1$, very low interest rates and the equilibrium is clearly inefficient.

Env. II – Storage

There is no trade, but households can store goods at fixed rate $1 + r > 0$.

Individual decision problem:

$$\max_{c_t(t), c_t(t+1)} \lambda u(c_t(t)) + (1-\lambda) u(c_t(t+1))$$

subject to

$$c_t(t) + s_t \leq y_1$$

$$c_t(t+1) \leq y_2 + (1+r) s_t$$

In equilibrium:

$$\left(\frac{\lambda}{1-\lambda}\right) \frac{u'(c_t(t))}{u'(c_t(t+1))} = 1 + r$$

$$c_t(t) + \frac{1}{1+r} c_t(t+1) = y_1 + \frac{1}{1+r} y_2$$

For $\lambda \to 0$, we get that $c_t(t) \to 0$ and $c_t(t+1) = (1+r)y_1 + y_2$.

Env. III – Transfers

Suppose we can carry out lump-sum transfers.

Consider the transfers $\tau_1(t) = -y_1$ and $\tau_2(t) = ny_1$.

These transfers are feasible in the aggregate:

$$N_t \tau_1(t) + N_{t-1} \tau_2(t) = N_{t-1} \left(-ny_1 + ny_1 \right) = 0,$$

i.e., we break even each period.

This yields the following consumption profile:

$$\begin{aligned} c_t(t) &= 0 \\ c_t(t+1) &= ny_1 + y_2 \end{aligned}$$

Which one is better? Storage or transfers? And why?

Efficiency in the OG Model Revisited

Suppose agents are very patient ($\lambda \to 0$) and only want to consume late.

Two options:

1. "Save" ... well store at a return $1 + r$.
2. Intergenerational redistribution.

Storing is better whenever $1 + r > n$, whereas intergenerational transfers are better whenever $1 + r < n$.

Conclusion:

When interest rate are sufficiently low, neither markets nor the households themselves can achieve efficiency.

What's going on here?

There is a special feature of postponing forever the costs of intergenerational transfers.

Hence, it is not the absence of missing markets here that matters, but the fundamental model structure.

The first-welfare theorem fails here almost by default and without any imperfections:

- we need a desire to shift resources forward in time
- we need a perpetual scheme that kicks the can down the road

Efficient Economies

For completeness, let's consider the opposite case where $\lambda \to 1$.

People value consumption only when young.

Here, the economy is *always* efficient and transfers cannot improve efficiency.

Why?

- Interest rates (or required rates of return on storage) would need to be high (i.e., $r \to \infty$) to induce savings.
- Transfers from the old to the young can never be Pareto-improving.